Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/PGDGI/SEM-1/DGI-105/2011-12

2011

FUNDAMENTALS OF CALCULUS, MATRICES, VECTORS & STATISTICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Very Short Answer Type Questions)

1. Answer any *ten* of the following questions very briefly : 10 × 1 = 10

   i) If the dependent variable y cannot be expressed explicitly in terms of x, then y is called an explicit / implicit function. State True/False.

   ii) $\frac{d}{dx}\left(\frac{u}{v}\right) = ?$

   iii) $\int \frac{dx}{x\sqrt{x^2-1}} = ?$

   iv) $\int uv\,dx = u\int v\,dx - \ldots\ldots ?$

v) The number N of children in a family, which can be counted but cannot be measured is a ..... variable.

vi) What is a class mark ?

vii) Define the rth moment aobut the mean $\bar{X}$.

viii) For mutually exclusive events,

$$p_r \{ E_1 + E_2 \} = ?$$

ix) If x and y be eleminated from the equations

$a_1 x + b_1 y = 0$ and $a_2 x + b_2 y = 0$, then

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = ?$$

x) What is a unit matrix ?

xi) A ..... is a quantity having both magnitude and direction.

xii) Define the cross or vector product.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. An electric lamp is at a height of 6 metres above the floor. An object falls freely under gravity starting from rest at the same height as the lamp but at a horizontal distance of $1\frac{1}{2}$ metres from it. Find the speed of the shadow of the object on the floor, when it has fallen through 4 metres.

3. Integrate $\int_0^2 \sqrt{4 - x^2}\, dx$.

4. In a company having 80 employees, 60 earn Rs. 25 per hour and 20 earn Rs. 30 per hour. (a) Determine the mean earnings per hour. (b) Would the answer in part (a) be the same if 60 employees earn a mean hourly wage of Rs. 25 per hour ? Prove your answer. $1\frac{1}{2} + 3\frac{1}{2}$

5. A car travels 25 miles at 25 mph, 25 miles at 50 mph and 25 miles at 75 mph. Find the arithmetic mean and the harmonic mean of the three velocities. Which is the correct velocity ?

6. Evaluate 50 !

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$ then find $\frac{dy}{dx}$.

   b) Examine the polynomial $x^5 - 5x^4 + 5x^3 - 1$ for maximum or minimum. 5 + 10

8. Integrate and prove that

$$\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin\left(bx - \tan^{-1}\frac{b}{a}\right) + k$$

9. Find the mean deviation of the heights of 100 male students at *XYZ* university (see table below) :

Height (in)	***Number of students***
60 — 62	5
63 — 65	18
66 — 68	42

69 — 71	27
72 — 74	8
	Total 100

10. a) If E_1 is the event "drawing an ace" from a deck of cards and E_2 is the event "drawing a spade", then find the probability of drawing either an ace or a spade or both.

b) Find the number of permutations of letters in the word "IMBIBE".

c) Find the number of combinations of the letters a, b, c, d taken two at a time.

d) Define Kurtosis and describe various types of peakedness of a distribution. 3 + 2 + 2 + 8

11. a) Evaluate the determinant $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$.

b) Give examples of following matrices along with relevant definition :

i) Null matrix

ii) Square matrix

iii) Diagonal matrix and

iv) Scalar matrix.

c) Given $A = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 4 & 3 \\ 0 & 6 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -2 & 3 \\ 1 & 1 & 0 \end{bmatrix}$

find $4A - 3B$. 5 + 6 + 4