# Name : <br> Roll No. : <br>  <br> Invigilator's Signature : <br> CS/M.Tech (ECE-VLSI)/SEM-1/MVLSI-101/2012-13 <br> <br> 2012 <br> <br> 2012 <br> ADVANCED ENGINEERING MATHEMATICS 

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

## GROUP - A

( Short Answer Type Questions )

1. Answer the following questions : $7 \times 2=14$
a) How many iterations do you need in bisection method to get the root if you start with $a=1$ and $b=2$ and the tolerance is $10^{-4}$ ?
b) If $f(z)=\frac{\sin ^{3} z}{z^{2012}}$ then find the poles and the order of the poles.
c) The Newton-Raphson method is used to find the root of the equation $x^{2}-2=0$. If the iteration started from -1 , then where the iteration will converge ?
d) A die is rolled three times. Find the probability that exactly one odd number turns up among the three outcomes.
e) Find the residue of the function $f(z)=\frac{\sin (z-1)}{(z-1)(z-2)}$ at $z=1$.
f) Find the points of local maxima and local minima of the function $f(x)=\sin 2 x$ in $[-\pi, \pi]$.
g) Evaluate $\bigoplus_{|z|=1} \frac{\sin z}{z(z-1)} \mathrm{d} z$.

## GROUP - B

## ( Long Answer Type Questions )

Answer any four of the following. $\quad 4 \times 14=56$
2. a) Show that $f(z)=|z|^{2}$ is continuous everywhere but it is nowhere differentiable except origin.
b) Evaluate $\int_{|z+i|=3} \frac{\sin z+\cos z}{z^{2}-1} \mathrm{~d} z$.
c) $\quad \int_{|z|=4} \frac{z}{(z-1)(z-2)^{2}} \mathrm{~d} z$ by Cauchy's integral formula. 4
3. a) Find a positive root of $x^{3}-3 x+1$ using method of bisection method.
b) Evaluate $\sqrt[3]{65}$ to three places of decimals by NewtonRaphson method.
c) Define order or convergence of an iterative method. Prove that bisection method is a linearly convergent. 4
4. a) State and prove Bayes' theorem.
b) An unbiased coin is tossed repeatedly until the outcome of two successive tosses is the same. Assuming that the trials are independent, find the expected number of tosses.

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c) The distribution function $F_{x}(x)=\left\{\begin{array}{l}0,-\infty<x<0 \\ \frac{1}{6}, 0 \leq x<1 \\ \frac{3}{6}, 1 \leq x<3 \\ 1, x \geq 3\end{array}\right.$

Find the values of $P(X=1)$ and $P(x=-1)$.
3
5. a) State and prove the Cauchy-Goursat theorem for complex valued function.
b) What is the probability that in a randomly chosen group of $r$ people, no people have the same birthday ? (Consider one year = 365 days). 4
c) Evaluate $\int_{i}^{2-i}(x y+i x) \mathrm{d} z$ along the straight line joining $z=i$ and $z=2-i$.
6. a) What is cubic spline ? Find the interpolating polynomial using cubic spline for the following data:

| $x$ | -1 | 0 | 1 |
| :--- | :---: | :--- | :---: |
| $Y=f(x)$ | 1 | 2 | -1 |

b) In a population of $N$ families, $50 \%$ of families have three children, $30 \%$ of the families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children ?

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7. a) Find the bilinear transformation, which maps the points $z=1,0,-1$ onto the points $w=i, 0,-i$ Also find the fixed points of the transformation.
b) Two identical urns contain respectively 7 white, 7 black balls and 4 white, 4 black balls. An urn is selected at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn ?
c) If $f(x)=\left\{\begin{array}{ll}k x(1-x), & 0<x<1 \\ 0 & \text { otherwise }\end{array}\right.$ is the density function of a continuous random variable $X$. Find the value of $k$ and the distribution function $F_{x}(x)$.

