



Name :

Roll No. :

Invigilator's Signature :

CS/M.TECH (ECE) VLSI/SEM-1/MVLSI-101/2010-11

2010-11

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Very Short Answer Type Questions)

1. Answer all of the following questions :

$$7 \times 2 = 14$$

- i) The Newton-Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If the iteration started from -1 , then where the iteration will converge ?
- ii) In method of Bisection for the equation $f(x) = 0$, $a \leq x \leq b$ find the length of the interval after n iteration.
- iii) Four fair coins are tossed simultaneously. Find the probability that at least one head and one tail turn up.
- iv) Find the residue of the function $f(z) = \frac{5z}{(z-1)(z-2)}$ at $z = 1$.



- v) If $3 \frac{dy}{dx} + 5y^2 = \sin x$, $y(0.3) = 5$, using a step size of $h = 0.3$, find the value of $y(0.9)$ using Euler's method.
- vi) Find the minimum value of the function $f(x) = e^x + e^{-x}$, where x is real number.
- vii) Evaluate $\oint_{|z|=1} \frac{\sin z}{z} dz$

GROUP – B

(Long Answer Type Questions)

Answer any *four* of the following questions :

$$4 \times 14 = 56$$

2. a) The velocity (m/s) of a body is given as a function of time (seconds) by

$$v(t) = 200 \ln(1 + t) - t, t \geq 0$$

Using Euler's method with a step size of 5 seconds, find the distance in metres travelled by the body from $t = 2$ to $t = 12$ seconds.

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- b) Evaluate $\sqrt{11}$ to three places of decimals by Newton-Raphson method.

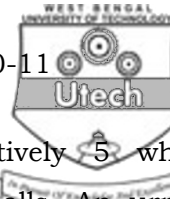
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- c) Prove that Newton-Raphson method has a quadratic convergence.

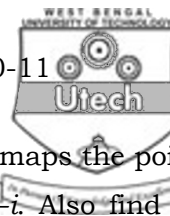
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3. a) State and prove Bayes theorem.

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- b) Two identical urns contains respectively 5 white, 7 black balls and 4 white, 2 black balls. An urn is selected at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn ? 5
- c) The distribution function $F_X(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{5}, & 0 \leq x < 1 \\ \frac{3}{5}, & 1 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$
- Find the values of $P(X = 1)$ and $P(X = -1)$. 3
4. a) Show that $f(z) = |z|^2$ is continuous everywhere but it is nowhere differentiable except origin. 5
- b) Evaluate $\oint_{|z|=4} \frac{z+1}{(z^2-2z)} dz$ 4
- c) Evaluate $\oint_{|z|=4} \frac{z}{(z-1)(z-2)^2} dz$ by residue theorem. 5
5. a) If $xyz = abc$, show that the maximum value of $bcx + cay + abz$ is $3abc$ by Lagrange's method of Multipliers. 7
- b) Show that $f(x, y, z) = (x + y + z)^3 - 3(x + y + z) - 24xyz$ has a minimum at $(1, 1, 1)$ and a maximum at $(-1, -1, -1)$. 7
6. a) Solve the recurrence relation $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$, $f_0 = 0$ and $f_1 = 1$. 5



- b) Find the bilinear transformation, which maps the points $z = 1, 0, -1$ onto the points $w = i, 0, -i$. Also find the fixed points of the transformation. 5
- c) The probability density function of a random variable X is $f(x) = k(x-1)(2-x)$ for $1 \leq x \leq 2$. Determine –
- the value of the constant k
 - $P\left(\frac{5}{4} \leq X \leq \frac{3}{2}\right)$. 4
7. a) Write the Kuhn-Tucker conditions for the following minimization problem : $Min. f(x) = x_1^2 + x_2^2 + x_3^2$, subject to $2x_1 + x_2 \leq 5, x_1 + x_3 \leq 2, x_1 \geq 1, x_2 \geq 2$ and $x_3 \geq 0$. 5
- b) Prove the condition of convergence for Newton-Raphson method to solve an transcendental equation $|f(x) \cdot f''(x)| \leq (f'(x))^2$. 4
- c) From the following table, find the value of $f(1.5)$ by Newton's forward interpolation formula or Lagrange's interpolation :

x	1	2	3	4	5
$f(x)$	5	10	15	20	25

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