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CS/M.TECH (ECE) VLSI/SEM-1/MVLSI-101/2010-11

2010-11

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours　　　　　　　　　　　　*Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Very Short Answer Type Questions)

1. Answer all of the following questions :

$7 \times 2 = 14$

i) The Newton-Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If the iteration started from -1, then where the iteration will converge ?

ii) In method of Bisection for the equation $f(x) = 0$, $a \le x \le b$ find the length of the interval after n iteration.

iii) Four fair coins are tossed simultaneously. Find the probability that at least one head and one tail turn up.

iv) Find the residue of the function $f(z) = \dfrac{5z}{(z-1)(z-2)}$ at $z = 1$.

v) If $3\frac{dy}{dx} + 5\ y^2 = \sin x$, $y\ (0.3) = 5$, using a step size of $h = 0.3$, find the value of $y\ (0.9)$ using Euler's method.

vi) Find the minimum value of the function $f(x) = e^x + e^{-x}$, where x is real number.

vii) Evaluate $\oint\limits_{|z|=1} \frac{\sin z}{z}\, dz$

GROUP – B

(Long Answer Type Questions)

Answer any *four* of the following questions :

$4 \times 14 = 56$

2. a) The velocity (m/s) of a body is given as a function of time (seconds) by

$$v(t) = 200 ln\,(1 + t) - t,\ t \geq 0$$

Using Euler's method with a step size of 5 seconds, find the distance in metres travelled by the body from $t = 2$ to $t = 12$ seconds. 5

b) Evaluate $\sqrt{11}$ to three places of decimals by Newton-Raphson method. 4

c) Prove that Newton-Raphson method has a quadratic convergence. 5

3. a) State and prove Bayes theorem. 6

b) Two identical urns contains respectively 5 white, 7 black balls and 4 white, 2 black balls. An urn is selected at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn ? 5

c) The distribution function $F_X(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{5}, & 0 \le x < 1 \\ \frac{3}{5}, & 1 \le x < 3 \\ 1, & x \ge 3 \end{cases}$

Find the values of P (X = 1) and P (X = – 1). 3

4. a) Show that $f(z) = |z|^2$ is continuous everywhere but it is nowhere differentiable except origin. 5

b) Evaluate $\oint_{|z|=4} \frac{z+1}{(z^2 - 2z)}\, dz$ 4

c) Evaluate $\oint_{|z|=4} \frac{z}{(z-1)(z-2)^2}\, dz$ by residue theorem. 5

5. a) If $xyz = abc$, show that the maximum value of $bcx + cay + abz$ is $3abc$ by Lagrange's method of Multipliers. 7

b) Show that $f(x, y, z) = (x+y+z)^3 - 3(x+y+z) - 24xyz$ has a minimum at (1, 1, 1,) and a maximum at (–1, –1, –1). 7

6. a) Solve the recurrence relation $f_n = f_{n-1} + f_{n-2}$ for $n \ge 2$, $f_0 = 0$ and $f_1 = 1$. 5

b) Find the bilinear transformation, which maps the points $z = 1, 0, -1$ onto the points $w = i, 0, -i$. Also find the fixed points of the transformation. 5

c) The probability density function of a random variable X is $f(x) = k(x-1)(2-x)$ for $1 \le x \le 2$. Determine –

i) the value of the constant k

ii) $P\left(\frac{5}{4} \le X \le \frac{3}{2}\right)$. 4

7. a) Write the Kuhn-Tucker conditions for the following minimization problem : *Min.* $f(x) = x_1^2 + x_2^2 + x_3^2$, subject to $2x_1 + x_2 \le 5$, $x_1 + x_3 \le 2$, $x_1 \ge 1$, $x_2 \ge 2$ and $x_3 \ge 0$. 5

b) Prove the condition of convergence for Newton-Raphson method to solve an transcendental equation $|f(x).f''(x)| \le (f'(x))^2$. 4

c) From the following table, find the value of f (1.5) by Newton's forward interpolation formula or Lagrange's interpolation :

x	1	2	3	4	5
$f(x)$	5	10	15	20	25

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