Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech(ECE-VLSI)/SEM-1/MVLSI-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Short Answer Type Questions)

1. Answer the following questions : 7 × 2 = 14

a) If $f(z) = \frac{\sin^2 z}{z^{2011}}$ then find the poles and the order of the poles.

b) The Newton-Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If the iteration started from -1, then where the iteration will converge.

c) In method of bisection for the equation

$f(x) = 0$ $a \le x \le b$ after n iteration (n starting from 1), find the length of the interval.

d) Three fair coins are tossed simultaneously. Find the probability that at least one head and one tail turn up.

e) Find the residue of the function

$$f(z) = \frac{5z}{(z-1)(z-2)} \text{ at } z = 1.$$

f) Find the minumum value of the function

$f(x) = 2011^{x} + 2011^{-x}$, where x is real number.

g) Evaluate : $\oint_{|z|=1} \frac{\cos z}{z(z-1)}\, dz$.

GROUP – B
(Long Answer Type Questions)

Answer any *four* of the following. $4 \times 14 = 56$

2. a) Show that $f(z) = |z|^2$ is continuous everywhere but it is no where differentiable except origin. 5

b) Evaluate $\int_{|z+i|=3} \frac{\sin z}{z^2 - 2iz - 10}\, dz$. 5

c) $\oint_{|z|=4} \frac{z}{(z-1)(z-2)^2}\, dz$ by Cauchy's integral formula. 4

3. a) Find a positive roots of $x^3 - 3x + 1$ using method of Regula-Falsi method. 5

b) Evaluate $\sqrt{11}$ to three places of decimals by Newton-Raphson method. 4

c) Prove that Newton-Raphson method has a quadratic convergence. 5

4. a) State and prove Bayes theorem. 6

b) Two identical urns contains respectively 2 white, 3 black balls and 4 white, 2 black balls, An urn is selected at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn ? 5

c) The distribution function $F_x(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{5}, & 0 \le x < 1 \\ \frac{3}{5}, & 1 \le x < 3 \\ 1, & x \ge 3 \end{cases}$

Find the value of $P(X = 1)$ and $P(x = -1)$. 3

5. a) State and prove the Cauchy-Goursat theorem for complex valued function. 6

b) Evaluate : $\oint_{|z|=2}^{-2} (z + z)\, dz$ 4

c) Evaluate : $\int_{i}^{2-i} (5xy + ix)\, dz$ along the straight line joining $z = i$ and $z = 2 - i$. 4

6. a) Find the probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7. 4

b) An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even numbered faces is the same. If the probability that the face is even given that it is greater than 3 is 0·75, find the probability that the face value exceeds 3. 5

c) In a population of N families, 50% of families have three children, 30% of the families have two children and the remaining families have one child. what is the probability that a randomly picked child belongs to a family with two children ? 5

7. a) If $xyz = abc$, show that the maximum value of $bcx + cay + abz$. 5

b) The probability density function of a random variable X is $f(x) = kx(x - 1)$ for $1 \leq x \leq 2$. Determine (i) the value of the constant k and (ii) $P\left(\frac{1}{2} \leq X \leq 1\right)$. 5

c) Prove that for any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$ 4

8. a) Solve the recurrence relations $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$, $a_0 = 0$ and $a_1 = 1$. 4

b) If Z_1, Z_2 and Z_3 are vertices of an equilateral triangle then show the

$$Z_1^2 + Z_2^2 + Z_3^2 = Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3$$ 5

c) Find all the solutions of $Z^{2011} = 1$ also interpret the solutions geometrically. 5