

CS/M.TECH (IT-SE)/SEM-1/MSE-104/2011-12 2011 DISCRETE STRUCTURE

Time Allotted: 3 Hours Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer any five questions. $5 \times 14=70$

1. a) Prove that a non-trivial finite ring having no divisor of zero is a ring with unity.
b) Define Integral Domain.
c) A commutative ring $R$ with unity is an integral domain iff for every non-zero element $a$ in $R$ a.u $=a . v \Rightarrow u=v$; where $u, v E R$. Prove.
2. a) Let $(G, \bullet)$ be an Abelian group and $H=\left\{a^{2} \mid a \in G\right\}$. Prove that H is a subgroup of G .
b) Does the set of all $2 \times 2$ non-singular matrices over integers form a group under matrix multiplication ? Justify.

3. a) A mapping $f: R \rightarrow \mathbb{R}$ is defined by $f(x)=x^{2}+x \in \mathbb{R}$. Show that $f$ is neither infective nor surjective. 5
b) Prove that the inverse of inverse of an element in a group $(G, \bullet)$ is the element itself.
c) Show that the following relation $R$ on $\mathbb{Z}$ is an equivalence relation $R=\left\{(\mathrm{a}, \mathrm{b}) ; \mathrm{a}, \mathrm{b} \in \mathbb{Z}\right.$ and $a^{2}+b^{2}$ is a multiple of 2$\}$.
4. a) Prove that a connected graph is a tree if and only if it has fewer edges than vertices.
b) Show that number of pendant vertices in a binary tree having $n$ vertices is $(n+1) / 2$.
c) Define level of a vertex in a binary tree and illustrate through example.
5. Use Ford-Fulkerson algorithm to find a maximal flow for the following network :

6. Find a shortest path from the vertex $a$ to the vertex $z$ by using Dijkstra's algorithm :

7. Find a minimal spanning tree for the following graph by using Prim's algorithm :

