



Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech (ME/SE/MT/MSS)/SEM-1/(ME/MMS/MM(ME)/SE(CE)-101)/2011-12

**2011**

**ADVANCED ENGINEERING MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

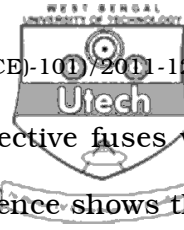
*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

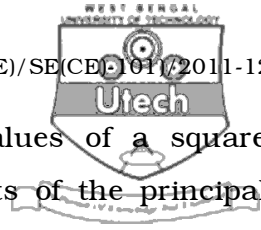
Answer any *five* questions.

5 × 14 = 70

1. a) Suppose 8% of the inhabitants of Kolkata are cricket fans.
  - i) Determine approximately the probability that 10 inhabitants chosen at random include at least 2 cricket fans.
  - ii) How many among 500 samples of 10 inhabitants each will contain at least 2 cricket fans ?
- b) In a bolt factory, machines A, B, C manufacture 25, 35 and 40 per cent of the total output respectively. If this output 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the product and is found to be defective, What is the probability that it was produced by A ?



2. a) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experience shows that 2% of such fuses are defective.
- b) If the weekly wages of 10,000 workers in a factory follows normal distribution with mean and s.d. Rs. 70 and Rs. 5 respectively, find the expected number of workers whose weekly wages are
  - i) between Rs. 66 and Rs. 72
  - ii) less than Rs. 66 and
  - iii) more than Rs. 72.
3. a) Find the Laplace transform of  $x e^{-x} \cos x$ .
- b) Find the Laplace transform of  $\left( \frac{1 - \cos 2t}{t} \right)$ .
4. a) By Newton-Raphson's method, find the root of  $x^4 - x - 10 = 0$ , which is near  $x = 2$ , correct upto 4-places of decimal.
- b) Solve the equation  $\frac{dy}{dx} = x + y$ , with initial condition  $y(0) = 1$  by Runge-Kutta method, from  $x = 0$  to  $x = 0.2$  with  $h = 0.2$ .



5. a) Show that the sum of the eigenvalues of a square matrix  $A$  is the sum of the elements of the principal diagonal.

b) If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , then show that  $A^2 - 4A - 5I = 0$ .

6. a) Determine by Power method the largest eigenvalue and the corresponding eigenvector of the following matrix :

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

- b) Show that, if a square matrix  $A$  has linearly independent eigenvectors, then a matrix  $P$  can be found such that  $P^{-1}AP$  is a diagonal matrix.

7. Solve any *three* of the following :

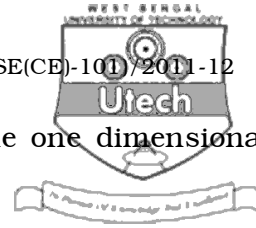
i)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$

ii)  $\frac{d^2y}{dx^2} + y = x^3$

iii)  $\frac{d^2y}{dx^2} - y = e^{2x}$

iv)  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = \sin 2x$

v)  $\frac{d^2y}{dx^2} = \frac{a}{y^3}$ .



8. a) Obtain the elementary solution of the one dimensional heat equation :

$$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \text{ where } k \text{ is a constant.}$$

- b) Solve by the method of separation of variables of the following P.D.E. :

$$\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

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