Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(EE-PS)/SEM-1/EMM-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and Question No. 2 as directed and any *three* from the rest.

1. Answer any *five* of the following : 5 × 2 = 10

   i) The system of equations

   $$3x + 2y = 3$$
   $$\frac{3}{2}x + y = 4$$

   is inconsistent. Justify.

   ii) Show that the eigenvalues of the matrix

   $$\begin{bmatrix} 0 & 3 \\ -4 & 1 \end{bmatrix}$$

   are all distinct and real.

   iii) Every real symmetric square martix is Hermitian. Justify.

iv) The function $\omega = \bar{z}$ is not analytic at $z = 0$. Substantiate your answer.

v) The function $Z = t\,\varphi(x)$ where φ is an arbitrary function is the solution of some partial differential equation. Show the equation.

vi) Is the function $g(t) = \int_0^t x^2\, dx$ a linear functional ? Justify.

vii) Show that $\{1, i\}$, $i = \sqrt{-1}$ is a basis of $\mathbb{C}$ and $\mathbb{R}$.

2. Answer any *five* of the following. $5 \times 3 = 15$

i) State Cayley-Hamilton theorem. If $A = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$, find its inverse by Cayley-Hamilton theorem.

ii) In the vector space $\mathbb{R}^3$, the vectors $v_1 = (1, 1, 0)$, $v_2 = (0, -1, 1)$ and $v_3 = (-1, 0, -1)$ are linearly dependent. Prove.

iii) Let V be a vector space and $S \leq V$; $L(S)$ is the linear span of S in V. Prove that $L(S)$ is a subpace of V.

iv) Find the complete integral of the equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = z^2$$

v) Prove that $\omega(z) = e^z$ is analytic at any point of the Z-plane.

vi) Evaluate the integral $\int_C \frac{dz}{z - z_0}$, C is a circle of radius r centred at the point z_0.

3. Derive Candy-Riemann equation for the function

$$\omega(z) = f(z),\ z = x + iy.$$

Are the conditions sufficient for the differentiability of $f(z)$? Justify your answer.

4. a) State and prove Cauchy's integral theorem.

b) Find the residues of the following function at its singular points :

$$f(z) = \frac{e^z}{(z+1)^3 (z-2)}.$$

5. Discuss the consistence of the following system of equations :

$$x + 2y + 2z = 1$$

$$2x + y + z = 2$$

$$3x + 2y + 2z = 3$$

$$y + z = 0$$

If possible, find a set of solutions thereof.

6. Find complete integral of any *three* of the following PDE : 3 × 5

a) $(y - z)p + (z - x)q = x - y$;

b) $\left(p^2 + q^2\right) y = qz$;

c) $px - qy = xz$;

d) $\left(x^2 - y^2 - z^2\right) p + 2xyq = 2xz;\ p \equiv \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$.

7. A tightly held string of length l fastened at both ends is disturbed from its equilibrium position by imparting to each of its points x an initial velocity $g(x)$. Show that the displacement at a distance x from one end at any time t is

$$y = \frac{2}{c\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\int_0^l g(x) \sin \frac{n\pi x}{l} \, dx \right) \sin \frac{n\pi x}{l} \sin \frac{nc\pi t}{l}$$

where c is a constant.

8. a) Find the eigenvalues and the corresponding eigen-vectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

b) Prove that to each eigenvector X of a square matrix A of order n, there corresponds a unique eigenvalue.

10 + 5