	Utech
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## CS/M.Tech(EE-PS)/SEM-1/EMM-101/2012-13 2012

## **ADVANCED ENGINEERING MATHEMATICS**

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and Question No. 2 as directed and any *three* from the rest.

- 1. Answer any *five* of the following :
- $5 \times 2 = 10$

i) The system of equations

$$3x + 2y = 3$$

$$\frac{3}{2}x + y = 4$$

is inconsistent. Justify.

ii) Show that the eigenvalues of the matrix

$$\begin{bmatrix} 0 & 3 \\ -4 & 1 \end{bmatrix}$$

are all distinct and real.

iii) Every real symmetric square martix is Hermitian.Justify.

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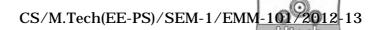


- iv) The function  $\omega = \overline{z}$  is not analytic at z = 0. Substantiate your answer.
- v) The function  $Z = t \varphi (x)$  where  $\varphi$  is an arbitrary function is the solution of some partial differential equation. Show the equation.
- vi) Is the function  $g(t) = \int_0^t x^2 dx$  a linear functional ?

  Justify.
- vii) Show that  $\{1, i\}$ ,  $i = \sqrt{-1}$  is a basis of  $\mathbb{C}$  and  $\mathbb{R}$ .
- 2. Answer any five of the following.

- $5 \times 3 = 15$
- i) State Cayley-Hamilton theorem. If  $A=\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$ , find its inverse by Cayley-Hamilton theorem.
- ii) In the vector space  $\mathbb{R}^3$ , the vectors  $v_1=(1,1,0)$ ,  $v_2=(0,-1,1)$  and  $v_3=(-1,0,-1)$  are linearly dependent. Prove.
- iii) Let V be a vector space and  $S \le V$ ; L(S) is the linear span of S in V. Prove that L(S) is a subpace of V.
- iv) Find the complete integral of the equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = z^2$$



- v) Prove that  $\omega$  ( z ) =  $e^z$  is analytic at any point of the Z-plane.
- vi) Evaluate the integral  $\int_{C} \frac{dz}{z-z_0}$ , *C* is a circle of radius

r centred at the point  $z_0$ .

3. Derive Candy-Riemann equation for the function

$$\omega(z) = f(z), z = x + iy.$$

Are the conditions sufficient for the differentiability of f(z)? Justify your answer.

- 4. a) State and prove Cauchy's integral theorem.
  - b) Find the residues of the following function at its singular points :

$$f(z) = \frac{e^{z}}{(z+1)^{3}(z-2)}$$
.

5. Discuss the consistence of the following system of equations:

$$x + 2y + 2z = 1$$

$$2x + y + z = 2$$

$$3x + 2y + 2z = 3$$

$$y + z = 0$$

If possible, find a set of solutions thereof.

6. Find complete integral of any three of the following PDE:

$$3 \times 5$$

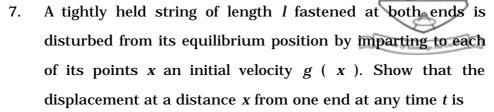
a) 
$$(y-z)p+(z-x)q=x-y$$
;

b) 
$$(p^2 + q^2)y = qz$$
;

c) 
$$px - qy = xz$$
;

d) 
$$(x^2 - y^2 - z^2) p + 2xyq = 2xz; p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$$
.

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$$y = \frac{2}{c\pi} \sum_{n=1}^{\infty} \frac{1}{n} \quad \left( \int_{0}^{1} g(x) \sin \frac{n\pi x}{l} dx \right) \sin \frac{n\pi x}{l} \sin \frac{nc\pi t}{l}$$

where c is a constant.

8. a) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\left[\begin{array}{cccc} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{array}\right]$$

b) Prove that to each eigenvector *X* of a square matrix *A* of order *n*, there corresponds a unique eigenvalue.

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