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CS/M.TECH (PBIR)/SEM-1/MBT/PHMB/PHMC-104/2012-13

2012

INTRODUCTORY MATHEMATICS

Time Allotted : 3 Hours　　　　　　　　　　　　Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

1. Answer any *ten* of the following : 　　　　$10 \times 1 = 10$

   i) Find the derivative of

   $$f(x) = \cos\left(ln\left(x^3 + 2x^2 + 5\right)\right).$$

   ii) Define the rank of a matrix.

   iii) Choose the correct answer :

   The function $f(x) = x^3 + x^2$ is

   a) even

   b) odd

   c) neither even nor odd.

iv) State *True* of *False* :

A polynomial function has no vertical or horizontal asymptoyes.

v) If $A = \begin{pmatrix} 1 & -4 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 3 \\ -2 & 0 \end{pmatrix}$, find the product AB.

vi) Choose the correct answer :

If $u = x^2 + y^2$, then $xu_x + yu_y$ is equal to

a) u

b) $2u$

c) $\frac{u}{2}$.

vii) Find the value of x for which the matrix $\begin{pmatrix} x & 4 \\ 3 & 2 \end{pmatrix}$ is singular.

viii) Write down the differential equation modelling the 'pure birth' process.

ix) Compute : $\int (x+1)\, e^{x^2+2x+1}\, dx$.

x) Define the term 'Wronskian'.

xi) Let $f(x) = 2x^2 + 1$, find an equation of the tangent line to the curve at the point (1, 3).

xii) Write down an example of the Bernoulli equation.

GROUP – B

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the equation :

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 0, \text{ when } x = 0, \; y = 3 \text{ and } \frac{dy}{dx} = 0.$$

3. If $V = \sqrt{x^2 + y^2 + z^2}$ show that

$$V_{xx} + V_{yy} + V_{zz} = \frac{2}{V}.$$

4. Find the eigenvalues of the matrix

$$A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}.$$

5. Compute : $\int e^x \cos x \, dx$.

6. Sketch the graph of the function : $y = \dfrac{x+1}{x-1}$.

GROUP – C

Answer any *three* of the following. $3 \times 15 = 45$

7\. a) If has been conjectured that a fish swimming a distance of L ft at a speed of v ft / sec. relative to the water and against a current flowing at the rate of u ft / sec ($u < v$) expends a total energy given by

$$E(v) = \frac{aLv^3}{v-u}$$

where E is measured in foot-pounds and a is a constant. Find the speed v at which the fish must swim in order to minimize the total energy expended.

b) The amount of nitrogen dioxide, a brown gas that impairs breathing, present in the atmosphere on a certain day in a city is approximated by

$$A(t) = \frac{136}{1 + 0\cdot25(t - 4\cdot5)^2} + 28, \quad (0 \le 1 \le 11)$$

where $A(t)$ is measured in Pollutant Standard Index (PSI) and t is measured in hours, with $t = 0$ corresponding to 7 a.m. Determine the time of day when the pollution is at its highest level.

c) If $y = a \ln x + bx^2 + x$ has its extreme values at $x = -1$ and $x = 2$, determine the values of a and b.

5 + 5 + 5

8. a) Find the critical point of the function

$$f(x, y) = 160x - 3x^2 - 2xy - 2y^2 + 120y - 18.$$

State whether $f(x, y)$ is maximum or minimum at this critical point. Suppose now that the constant $x + y = 35$ is imposed. Use the method of Lagrange Multiplier to determine the new critical point of the function $f(x, y)$ subject to the constraint mentioned above.

b) A parasitoid is an organism that attaches to or within a host during part of their development. Unlike parasites, parasitoids ultimately kill their hosts. The Nicholson-Bailey model is a frequently used model to describe the population dynamics of the host-parasitoid system, in which it is assumed that the number of parasitized hosts, denoted by N_a is given by

$$N_a = N\left[1 - e^{-bP}\right]$$

where N is the host density, P is the parasitoid density and b is the searching efficiency of the parasitoid. Express b as a function of P, N and N_a. Evaluate $\frac{\partial b}{\partial P}$ and discuss how b is affected when P increases.

c) Solve : $x^2 y'' - 2 \cdot 5\, xy' - 2 \cdot 0 y = 0$ 6 + 5 + 4

9. a) Solve the equation :

$$(x + y + 1)\, dx - (2x + 2y + 1)\, dy = 0.$$

b) Solve the equation :

$$y'' + 2y' + y = e^{-x}, \quad y(0) = -1, \quad y'(0) = 1.$$

c) Consider a predator-prey system made up of a single predator and a single prey. Suppose that a some instant t, the prey population is x and the predator population is y. Under suitable assumptions (which you should state) set up the differential equations modeling the change of the predator and the prey populations taking into account the predator-prey interactions. 4 + 6 + 5

10. a) Semelparous organisms breed only once during their lifetime. Examples of this type of reproduction strategy can be found with Pacific salmon and bamboo. The per capita rate of increase, r, can be thought of as a measure of reproductive fitness. The greater r, the more offspring an individual produces. The intrinsic rate of increase is typically a function of age x. Models for age-structured populations of semelparous organisms predict that the intrinsic rate of increase as a function of x is given by

$$r(x) = \frac{\ln [\, l(x)\, m(x) \,]}{x}$$

where $l(x)$ is the probability of surving to age x and $m(x)$ is the number of female births at age x. Suppose that $l(x) = e^{-ax}$, $m(x) = bx^{c}$ where a, b, c are positive constants. Find the optimal age of reproduction.

b) Let $Y(t)$ be a population of yeast in a sugar solution that begins with a concentration of 10 yeast/ml. If the concentration of yeast is given by $Y(t) = 10e^{at}$, then find the value of a assuming that the concentration doubles every 2 hours. Also find the rate of increase in the concentration of yeast per hour. 9 + 6

11. a) Determine the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.

b) Evaluate the following determinant by suitable reduction :

$$\begin{vmatrix} 2 & 0 & -4 & 6 \\ 4 & 5 & 1 & 0 \\ 0 & 2 & 6 & -1 \\ -3 & 8 & 9 & 1 \end{vmatrix}$$

8 + 7