



Name :

Roll No. :

Invigilator's Signature :

CS/M.TECH (PBIR)/SEM-1/MBT/PHMB/PHMC-104/2012-13

2012

INTRODUCTORY MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

1. Answer any *ten* of the following : 10 × 1 = 10

i) Find the derivative of

$$f(x) = \cos \left(\ln \left(x^3 + 2x^2 + 5 \right) \right).$$

ii) Define the rank of a matrix.

iii) Choose the correct answer :

The function $f(x) = x^3 + x^2$ is

- a) even
- b) odd
- c) neither even nor odd.



iv) State *True* or *False* :

A polynomial function has no vertical or horizontal asymptotes.

v) If $A = \begin{pmatrix} 1 & -4 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 3 \\ -2 & 0 \end{pmatrix}$, find the product AB .

vi) Choose the correct answer :

If $u = x^2 + y^2$, then $xu_x + yu_y$ is equal to

a) u

b) $2u$

c) $\frac{u}{2}$

vii) Find the value of x for which the matrix $\begin{pmatrix} x & 4 \\ 3 & 2 \end{pmatrix}$ is singular.

viii) Write down the differential equation modelling the 'pure birth' process.

ix) Compute : $\int (x + 1) e^{x^2 + 2x + 1} dx$.



- x) Define the term 'Wronskian'.
- xi) Let $f(x) = 2x^2 + 1$, find an equation of the tangent line to the curve at the point $(1, 3)$.
- xii) Write down an example of the Bernoulli equation.

GROUP - B

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the equation :

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 0, \text{ when } x = 0, y = 3 \text{ and } \frac{dy}{dx} = 0.$$

3. If $V = \sqrt{x^2 + y^2 + z^2}$ show that

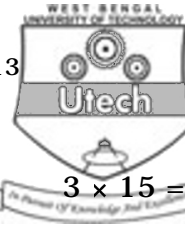
$$V_{xx} + V_{yy} + V_{zz} = \frac{2}{V}.$$

4. Find the eigenvalues of the matrix

$$A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}.$$

5. Compute : $\int e^x \cos x \, dx$.

6. Sketch the graph of the function : $y = \frac{x+1}{x-1}$.

**GROUP - C**Answer any *three* of the following. $3 \times 15 = 45$

7. a) It has been conjectured that a fish swimming a distance of L ft at a speed of v ft/sec. relative to the water and against a current flowing at the rate of u ft/sec ($u < v$) expends a total energy given by

$$E(v) = \frac{aL v^3}{v - u}$$

where E is measured in foot-pounds and a is a constant. Find the speed v at which the fish must swim in order to minimize the total energy expended.

- b) The amount of nitrogen dioxide, a brown gas that impairs breathing, present in the atmosphere on a certain day in a city is approximated by

$$A(t) = \frac{136}{1 + 0.25(t - 4.5)^2} + 28, \quad (0 \leq t \leq 11)$$

where $A(t)$ is measured in Pollutant Standard Index (PSI) and t is measured in hours, with $t = 0$ corresponding to 7 a.m. Determine the time of day when the pollution is at its highest level.

- c) If $y = a \ln x + bx^2 + x$ has its extreme values at $x = -1$ and $x = 2$, determine the values of a and b .

5 + 5 + 5



8. a) Find the critical point of the function

$$f(x, y) = 160x - 3x^2 - 2xy - 2y^2 + 120y - 18.$$

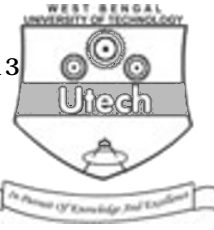
State whether $f(x, y)$ is maximum or minimum at this critical point. Suppose now that the constant $x + y = 35$ is imposed. Use the method of Lagrange Multiplier to determine the new critical point of the function $f(x, y)$ subject to the constraint mentioned above.

- b) A parasitoid is an organism that attaches to or within a host during part of their development. Unlike parasites, parasitoids ultimately kill their hosts. The Nicholson-Bailey model is a frequently used model to describe the population dynamics of the host-parasitoid system, in which it is assumed that the number of parasitized hosts, denoted by N_a is given by

$$N_a = N [1 - e^{-bP}]$$

where N is the host density, P is the parasitoid density and b is the searching efficiency of the parasitoid. Express b as a function of P , N and N_a . Evaluate $\frac{\partial b}{\partial P}$ and discuss how b is affected when P increases.

- c) Solve : $x^2 y'' - 2.5 xy' - 2.0y = 0$ 6 + 5 + 4



9. a) Solve the equation :

$$(x + y + 1) dx - (2x + 2y + 1) dy = 0.$$

- b) Solve the equation :

$$y'' + 2y' + y = e^{-x}, \quad y(0) = -1, \quad y'(0) = 1.$$

- c) Consider a predator-prey system made up of a single predator and a single prey. Suppose that at some instant t , the prey population is x and the predator population is y . Under suitable assumptions (which you should state) set up the differential equations modeling the change of the predator and the prey populations taking into account the predator-prey interactions.

4 + 6 + 5

10. a) Semelparous organisms breed only once during their lifetime. Examples of this type of reproduction strategy can be found with Pacific salmon and bamboo. The per capita rate of increase, r , can be thought of as a measure of reproductive fitness. The greater r , the more offspring an individual produces. The intrinsic rate of increase is typically a function of age x . Models for age-structured populations of semelparous organisms predict that the intrinsic rate of increase as a function of x is given by

$$r(x) = \frac{\ln [l(x) m(x)]}{x}$$

where $l(x)$ is the probability of surviving to age x and $m(x)$ is the number of female births at age x . Suppose that $l(x) = e^{-ax}$, $m(x) = bx^c$ where a, b, c are positive constants. Find the optimal age of reproduction.



- b) Let $Y(t)$ be a population of yeast in a sugar solution that begins with a concentration of 10 yeast/ml. If the concentration of yeast is given by $Y(t) = 10e^{at}$, then find the value of a assuming that the concentration doubles every 2 hours. Also find the rate of increase in the concentration of yeast per hour. 9 + 6

11. a) Determine the eigenvalues and eigenvectors of the

matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.

- b) Evaluate the following determinant by suitable reduction :

$$\begin{vmatrix} 2 & 0 & -4 & 6 \\ 4 & 5 & 1 & 0 \\ 0 & 2 & 6 & -1 \\ -3 & 8 & 9 & 1 \end{vmatrix}$$

8 + 7
