Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech (BT)/Int.Ph.D (Mol.Bio-Micro.Bio)/SEM-1/MBT-104/PHMB-104/PHMC-104/2010-11

2010-11
INTRODUCTORY MATHEMATICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Objective Type Questions)

1. Answer any *ten* of the following questions : $10 \times 1 = 10$

i) Find the derivative of $f(x) = \sin\left(ln\left(2x^2 + 5\right)\right)$.

ii) Find $\frac{dy}{dx}$ at $x = -2$, if $y = x^3 - 5x^2 + 4x + 2$.

iii) The curve of the $f(x) = x + (1/x)$ is symmetric with respect to

a) the y-axis b) the x-axis

c) the origin d) none of these.

iv) The equation $y = (x + 2)/(x + 5)$ represents a ....

v) Find A^{-1} if $A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$.

vi) Matrix multiplication is a .... operation.

vii) Write down the differential equation modelling the "pure birth" process.

viii) Write down an example of a non-linear ordinary differential equation.

ix) Compute : $\int_0^{\frac{\pi}{2}} \log \tan x \, dx$.

x) Consider the matrices $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. Calculate $\left(A^2 + B^2\right)$.

xi) What is the domain of the function $f(x) = \sqrt{\left(x^2 - 5x + 6\right)}$?

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the equation : $\frac{dy}{dx} + 2y = 2$, y (0) = 2.

3. Consider the function :

$$f(x) = \frac{1}{2} - x, \text{ when } 0 < x < \frac{1}{2}$$
$$= \frac{1}{2}, \text{ when } x = \frac{1}{2}$$
$$= \frac{3}{2} - x, \text{ when } \frac{1}{2} < x < 1$$

Test the continuity of the function $f(x)$ at $x = \frac{1}{2}$.

4. Find the eigenvalues of the matrix $A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$.

5. Compute : $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$.

6. Find the maximum an minimum values of the function $f(x) = x^3 - 3x^2 - 6x + 1$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Nutrients in low concentrations inhibit growth of an organism, but high concentrations are often toxic. Let c be the concentration of a particular nutrient (in moles/liter) and P be the population density of an organism (in number/sq. cm). Suppose that it is found that the effect of this nutrient cause the population to grow according to the equation : $P(c) = 1000\,c / \left(1 + 100\,c^2\right)$.

   i) Find the concentration of the nutrient that yields the largest population density of this organism and calculate the population density of this organism at this optimal concentration.

   ii) Sketch a graph of the population density of this organism as a function of the concentration of the nutrient.

   b) Sketch the function $y = (2x + 1)/(x - 2)$, showing the asymptotes, if any. 9 + 6

8. a) Consider the function $f(x) = x + 1/x$. Show that the value of the function at its (local) minimum is greater than the value at its (local) maximum. Identify the asymptotes and sketch the curve of this function.

   b) Find the critical point of the function $f(x, y) = 160x - 3x^2 - 2xy - 2y^2 + 120y - 18$. How does this critical point change when the constraint $x + y = 35$ is imposed ? 9 + 6

9. a) Solve the equation :

$y'' + 2y' + y = e^{-x}$, y (0) = – 1, $y'(0) = 1$.

b) Solve the equation :

$x^2 y'' - 2 \cdot 5 xy' - 2 \cdot 0 y = 0$.

c) State the Mean Value Theorem. 8 + 4 + 3

10. a) A mammalian cell line is susceptible to infection by virus like particles but can serve as host for only one such particle. The rate of infection equals half the number of uninfected cells, and the rate of elimination of the particles is half the number of infected particles. Set up a differential equation giving dI/dt in terms of C and I, where I is the number of infected particles and C is the total number of cells. The number of cells is growing at a steady rate, so that $C = C_0 + 10t$, where C_0 is the initial number of cells. Solve the differential equation and show that eventually half the cells will be infected.

b) Let $Y(t)$ be a population of yeast in a sugar solution that begins with a concentration of 10 yeast/ml. If the concentration of yeast is given by $Y(t)$ = $10e^{at}$, then find the value of a assuming that the concentration doubles every 2 hours. Also find the rate of increase in the concentration of yeast per hour. 9 + 6

11. a) Find the eigenvalues of the matrix $A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}$.

Hence determine the eigenvectros corresponding to the smallest and the largest eigenvalue.

b) Solve the following differential equation by the method of variation of parameters : $y'' - 4y' + 4y = e^{2x}/x$. 9 + 6