Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(ME)/SEM-1/MM(ME)-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any ***five*** questions. $5 \times 14 = 70$

1. a) An incomplete distribution is given below :

Class	0- 10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	10	20	?	40	?	25	15	170

The median is 35. Find the missing frequencies.

b) Given the following bivariate data :

x	1	5	3	2	1	1	7	3
y	6	1	0	0	1	2	1	5

Fit the regression line of y on x and that of x on y. Predict y when $x = 10$ and x when $y = 2 \cdot 5$. 7 + 7

2. a) During a countrywide investigation the incidence of TB was found to be 1%. In a college of 400 strength 5 were reported to be affected whereas in another college of 1200 strength 10 were reported to be affected. Does this indicate any significant difference ?

b) Use the method of least squares to fit a parabola to the following data :

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

7 + 7

3. a) Let $x_0, x_1, \ldots, x_n$ be $n + 1$ distinct points in the interval $[\, a, b \,]$ and let $y_0, y_1, \ldots, y_n$ be any set of $n + 1$ real numbers. Then show that there exist a unique polynomial $p(x)$ in p_n such that $p(x_j) = y_j$ for $0 \le j \le n$.

b) If $y(1) = -3$, $y(3) = 9$, $y(4) = 30$ and $y(6) = 132$, find the interpolation polynomial that takes the same values as the function y at the given points. 7 + 7

4. a) Determine the largest eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ correct to two decimal places using power method.

b) Solve the following system of equations, correct to 2 decimal places, by Newton-Raphson method with $(1, 2)$ as initial approximation :

$\sin xy + x - y = 0$

$y \cos xy + 1 = 0$ 7 + 7

5. Solve the boundary value problem :

$y'' + y = x\,,\ 0 < x < 1$

$y\,(0) = 0\,,\ y\,(1) = 0$

by Rayleigh-Ritz method using the approximation function $W(x) = x\,(1-x)(a_1 + a_2 x)$. 14

6. a) A semi-infinite solid $x > 0$ is initially at temperature zero. At time $t > 0$, a constant temperature u_0 is applied and maintained at the face $x = 0$. Use Laplace transform technique to find the temperature at any point of the solid at any time $t > 0$.

b) Use Laplace transform to solve the initial value problem

$\left[D^2 + tD - 1\right] y = 0$

$y\,(0) = 1\,,\ y'(0) = 1$

where D stands for $\dfrac{d}{dt}$. 7 + 7

7. a) A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form $y = \lambda \sin \dfrac{\pi x}{L}$ from which it is released from rest at time $t = 0$. Find the displacement of any point on the string at any time t by the method of separation of variables.

b) The faces of a thin rectangular copper plate of sides a and b are perfectly installed. The temperature equals a specified function $f\,(\,x\,)$ on the lower side and 0 on the other three sides of the plate. Find the steady state temperature $u\,(\,x,\ y\,)$ in the plate by the method of separation of variables. 7 + 7

8. a) Find the Fourier Transform of $f(x) = e^{-a^2 x^2}$.

b) Express the function $f(x) = 1,\ 0 \le x \le \pi$

$= 0, x > \pi$

as a Fourier cosine Integral. Hence evaluate

$\int_0^\infty \frac{\cos px \sin p\pi}{p} \, dp$. 7 + 7

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