Name : ....

Roll No. : ....

Invigilator's Signature : ....

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. $5 \times 14 = 70$

1. a) Suppose 8% of the inhabitants of Kolkata are cricket fans.

   i) Determine approximately the probability that 10 inhabitants chosen at random include at least 2 cricket fans.

   ii) How many among 500 samples of 10 inhabitants each will contain at least 2 cricket fans ?

   b) In a bolt factory, machines *A*, *B*, *C* manufacture 25, 35 and 40 per cent of the total output respectively. If this output 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the product and is found to be defective, What is the probability that it was produced by *A* ?

2. a) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experience shows that 2% of such fuses are defective.

b) If the weekly wages of 10,000 workers in a factory follows normal distribution with mean and *s.d.* Rs. 70 and Rs. 5 respectively, find the expected number of workers whose weekly wages are

i) between Rs. 66 and Rs. 72

ii) less than Rs. 66 and

iii) more than Rs. 72.

3. a) Find the Laplace transform of $x\,e^{-x}\cos x$.

b) Find the Laplace transform of $\left(\dfrac{1-\cos 2t}{t}\right)$.

4. a) By Newton-Raphson's method, find the root of $x^4 - x - 10 = 0$, which is near $x = 2$, correct upto 4-places of decimal.

b) Solve the equation $\dfrac{dy}{dx} = x + y$, with initial condition $y(0) = 1$ by Runge-Kutta method, from $x = 0$ to $x = 0{\cdot}2$ with $h = 0{\cdot}2$.

5. a) Show that the sum of the eigenvalues of a square matrix A is the sum of the elements of the principal diagonal.

b) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I = 0$.

6. a) Determine by Power method the largest eigenvalue and the corresponding eigenvector of the following matrix :

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

b) Show that, if a square matrix A has linearly independent eigenvectors, then a matrix P can be found such that $P^{-1}AP$ is a diagonal matrix.

7. Solve any *three* of the following :

i) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 5y = 0$

ii) $\dfrac{d^2y}{dx^2} + y = x^3$

iii) $\dfrac{d^2y}{dx^2} - y = e^{2x}$

iv) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = \sin 2x$

v) $\dfrac{d^2y}{dx^2} = \dfrac{a}{y^3}$.

8. a) Obtain the elementary solution of the one dimensional heat equation :

$$k \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \text{ where } k \text{ is a constant.}$$

b) Solve by the method of separation of variables of the following P.D.E. :

$$\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$