

Name :

Roll No. :

Invigilator's Signature :

CS/M.Tech (ME)/SEM-1/PTM-103/2009-10

2009

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions.

5 × 14 = 70

1. a) The mean monthly salary paid to all employees in a certain company was rupees 500. The mean monthly salaries paid to male and female employees were 520 and 420 rupees respectively. Obtain the percentage of male to female employees in the company.
- b) Given that the mode of the following frequency distribution of 70 students is 58.75. Find the missing frequencies f_1 and f_2 .

| | | | | |
|------------------|---------|---------|---------|---------|
| Class-interval : | 52 - 55 | 55 - 58 | 58 - 61 | 61 - 64 |
| Frequency : | 15 | f_1 | 25 | f_2 |

7 + 7

2. a) Prove that the standard deviation is independent of any change of origin but is dependent on the change of scale.

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[Turn over



- b) Find the standard deviation of the first n natural numbers.
- c) The scores of two batsmen A and B in 10 innings during a certain season are as follows :

| | | | | | | | | | | |
|-----|----|----|----|----|----|----|----|----|----|----|
| A : | 32 | 28 | 47 | 63 | 71 | 39 | 10 | 60 | 96 | 14 |
| B : | 19 | 31 | 48 | 53 | 67 | 90 | 10 | 62 | 40 | 80 |

Find which of the two batsmen A and B is more consistent in scoring.

3 + 3 + 8

3. a) Determine the constants a and b by the method of least squares such that $y = ae^{bx}$ fits the following data :

| | | | | | |
|-------|-------|--------|--------|--------|--------|
| $x :$ | 2 | 4 | 6 | 8 | 10 |
| $y :$ | 4.077 | 11.084 | 30.128 | 81.897 | 222.62 |

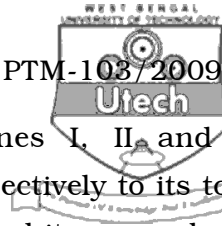
- b) From the following data obtain the two regression lines and correlation coefficient :

| | | | | | | | |
|-------|-----|----|----|----|-----|----|----|
| $x :$ | 100 | 98 | 78 | 85 | 110 | 93 | 80 |
| $y :$ | 85 | 90 | 70 | 72 | 95 | 81 | 74 |

Find the value of y when $x = 82$.

7 + 7

4. a) If A and B are independent events then prove that \bar{A} and \bar{B} are also independent events.
- b) The chance of X solving a particular problem is $\frac{2}{3}$ and Y solving the same problem is $\frac{3}{4}$. What is the probability that the problem is solved if they try independently ?



- c) A factory has three production lines I, II, and III contributing 20%, 30% and 50% respectively to its total output. The percentages of substandard items produced by lines I, II and III are respectively 15, 10 and 2. If an item chosen at random from the total output is found to be substandard, what is the probability the item is from line I ? 4 + 4 + 6

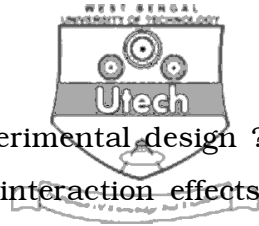
5. a) X is discrete random variable having the following probability mass function :

| | | | | | | | | |
|-----------------------|---|-----|------|------|------|-------|--------|------------|
| Mass points (x) : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $P (X = x)$ | 0 | k | $2k$ | $2k$ | $3k$ | k^2 | $2k^2$ | $7k^2 + k$ |

- i) Determine the constant k .
 - ii) Find $P (X < 6)$
 - iii) What will be $P (X \geq 6)$?
- b) Find the mean and standard deviation of binomial distribution with parameters n and p . 6 + 8
6. a) The marks of 1000 students in a university are found to be normally distributed with mean 70 and s.d. 5. Estimate the number of students whose marks will be
- i) between 60 and 75
 - ii) more than 75
 - iii) less than 68

[Given that $P (0 < Z < 1) = 0.3413$,

$P (0 < Z < 2) = 0.4772$, $P (0 < Z < 0.4) = 0.1554$]



- b) What are the basic principles of experimental design ?
Define the terms 'main effects' and 'interaction effects' in relation to a 2^2 -experiment. 6 + (2 + 6)

7. a) Describe Gaussian elimination method for numerical solution of a system of linear equation. What is the necessity of the pivoting process involved in it ?

- b) Solve the following system of equations by Gauss-Seidel's method correct to 3 decimal places :

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

$$x + y + 54z = 110$$

8. a) Solve the following system of equations by LU decomposition method :

$$x + y + z = 1$$

$$4x + 3y - z = 6$$

$$3x + 5y + 3z = 4 \quad (5 + 2) + 7$$

- b) Determine the largest eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$ correct to 2 decimal places using the power method. 7 + 7
