Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH (ECE-COMM)/SEM-2/MCE-202/2012

2012
ERROR CONTROL AND CODING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct answers for any *ten* of the following : $10 \times 1 = 10$

i) A linear code C of minimum distance d_{min} can detect upto t errors if and only if

a) $d_{min} \geq t + 1$ b) $d_{min} \leq t + 1$

c) $d_{min} \geq 2t + 1$ d) $d_{min} \leq 2t + 1$.

ii) For BCH code, block length, n is equal to

a) $2^m - 1$ b) 2^m

c) $2^m + 1$ d) $\{ 2^m - 1)/2 \}$.

iii) Relationship among information rate, message rate at entropy is

a) $r = RH$ b) $R = rH$

c) $R = r^2 H$ d) $R = r^3 H$.

iv) If the two codes have same set of codewords, code is known as

a) Galoy code b) Hoffmann code

c) Self-dual code d) Dual code.

v) Field elements that can generate all the non-zero elements of a field are said

a) Galois field b) Shannon-Fano Code

c) Primitive d) Cyclic code.

vi) Given the vectors $u = (2,7,1)$ and $v = (10,-3,8)$, value of $(24 - 6v)$ is equal to

a) (– 66, 35, – 54) b) (– 66, 30, 54)

c) (60, 30, 10) d) (10, 20, 30).

vii) Polynomial that is divisible only by itself and 1 are referred as

a) Reducible Polynomial

b) Irreducible Polynomial

c) Primitive Polynomial

d) Prime Polynomial.

viii) Inverse of the matrix, $A = \begin{bmatrix} \alpha^3 & \alpha^5 \\ 1 & \alpha \end{bmatrix}$ is

a) $\begin{bmatrix} 1 & \alpha^5 \\ \alpha & \alpha^3 \end{bmatrix}$ b) $\begin{bmatrix} \alpha^5 & 1 \\ \alpha^3 & \alpha \end{bmatrix}$

c) $\begin{bmatrix} \alpha & \alpha^5 \\ 1 & \alpha^3 \end{bmatrix}$ d) $\begin{bmatrix} \alpha & 1 \\ \alpha^5 & \alpha^3 \end{bmatrix}$.

ix) The minimal polynomial of α^5 is

a) $x^3 + x + 1$ b) $x^3 + 1$

c) $x^2 + 1$ d) $x^3 + x^2 + 1$.

x) In Block Code, syndrome is represented by

a) rA^T b) r/A^T

c) rH d) $H^2 r$.

xi) Consider the following code vectors :

$C_1 = [\ 1\ 0\ 0\ 1\ 0\ ]$; $C_2 = [\ 0\ 1\ 1\ 0\ 1\ ]$; $C_3 = [\ 1\ 1\ 0\ 0\ 1]$.

Value of d (C_1, C_3) is

a) 2 b) 0

c) 3 d) 1.

xii) $C = [000, 001, 101]$ is not a

a) cyclic b) linear

c) Galoy d) convolution.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Construct the group of integers under modulo-6 addtition.

3. a) Determine whether the following sets form multiplicative groups :

   i) The set of real numbers

   ii) The set of positive real numbers

   iii) The set of negative real numbers.

   b) Show that the modulo-8 multiplication over {1,2,3,4,5,6,7} fails to form multiplicative group. 3 + 2

4. Construct a single-error-correcting binary BCH code over GF (2^3).

5. Show that

   a) $x^4 + x^2 + x + 1 = 0$ modulo-$(x + 1)$

   b) $R_{(x^2 + x + 1)}[x^5 + x^3 + x^2 + x + 1] = x$ 3 + 2

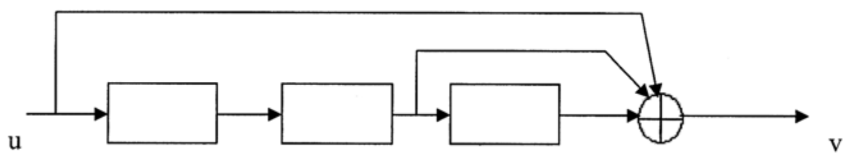
6. Draw an encoder for the (2,1,3) convolutional code with generator sequences $g^{(1)} = (1\ 0\ 1\ 1)$ and $g^{(2)} = (1\ 1\ 1\ 1)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Determine the generator sequences of the shift register as shown below :



Hence using convolution, determine the output sequences when the input sequences are

i) (1 1 0 1 ....) ;

ii) (1 1 0 1 1 1)

b) A parity-check code has the parity-check matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) Determine the generator matrix G

ii) Find the code word that begins with 1 0 1 ......

iii) Suppose that the received word is 1 1 0 1 1 0. Decode this received word. 7 + 8

8. a) For a (6, 3) systematic linear block code, the three parity-check bits C_4, C_5 and C_6 are formed from the following equations :

$$C_4 = d_1 + d_3$$

$$C_5 = d_1 + d_2 + d_3$$

$$C_6 = d_1 + d_2$$

i) Write down the generator matrix G.

ii) Construct all possible code words.

iii) Suppose that the received word is 010111.

b) Show that all error vectors that differ by a code have the same syndrome. 9 + 6

9. a) Determine whether the following sets form additive groups :

i) $S_1 = \{\pm 1, \pm 2, \pm 3, \ldots\ldots\}$

ii) $S_2 = \{0, 1, 2 \ldots\ldots\}$

iii) $S_3 = \{0, \pm 2, \pm 4, \ldots\ldots\}$

iv) $S_4 = \{0, \pm 1, \pm 3, \ldots\ldots\}$

b) Show that the set of vectors (0 0 0 0 0), (0 1 0 1 0), (1 1 0 0 1), (1 0 0 1 1) and (1 0 1 1 0) does not form a vector subspace of V_5.

c) Determine whether the vectors (0 1 1 0 1 1), (1 1 0 1 1 0), (0 1 1 1 1 0) and (1 1 0 0 1 1) are linearly independent. 5 + 5 + 5

10. a) Construct a (7, 4) cyclic code with $g(x) = 1 + x + x^3$.

i) Let the data word d = (1 0 1 0). Find the corresponding code word.

ii) Let the code word c = (1 1 0 0 1 0 1). Find the corresponding data word.

b) Let $g(x)$ be the generator polynomial of a cyclic code C. Find a scheme for encoding the data sequence $(d_0, d_1, \ldots\ldots, d_{k-1})$ into an (n, k) systematic code C.

c) Let C be a (7, 4) cyclic code with $g(x) = 1 + x + x^3$. Find a generator matrix G for C and find the code word for $d = (1\ 0\ 1\ 0)$. 5 + 5 + 5

11. a) Given the (7, 3) linear code with generator matrix

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Determine a systematic form of G. Hence find a parity-check matrix for the code.

b) Given that $C = (0\ 1\ 1\ 0\ 0\ 0\ 1\ 1)$ is a code word of (8, 4) extended Hamming code with even parity. Determine the decoding decisions when C incurs the errors

i) $e_1 = (0\ 0\ 0\ 0\ 0\ 1\ 0\ 0)$

ii) $e_2 = (0\ 1\ 0\ 1\ 0\ 0\ 0\ 0)$

iii) $e_3 = (0\ 0\ 0\ 1\ 1\ 0\ 1\ 0)$. 7 + 8