	Utech
Name:	
Roll No.:	Construction of State
Invigilator's Signature:	

CS/M.TECH/MBIN/SEM-1/MBIN-104/2012-13 2012

MATHEMATICS AND STATISTICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *three* questions from **Group-A** and any *four* questions from **Group-B**.

1. Answer any ten questions :

 $10 \times 1 = 10$

a) Show that the following system of equations has infinite number of solutions :

$$x + 2y + 3z = 1$$
, $3x + 4y + 5z = 2$, $5x + 6y + 7z = 3$.

b) If
$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -3 & 0 \end{pmatrix}$$
, $B = \begin{pmatrix} 7 & 9 & 0 \\ 1 & 1 & 5 \end{pmatrix}$ find matrix

C, such that A + B + C = O, where O is the null matrix of appropriate order.

40972 Turn over



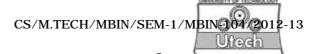
c) If
$$A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$$
, prove that $A^2 - 3I = 0$, where

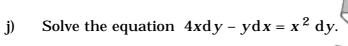
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

- d) Find the adjoint of $\begin{pmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{pmatrix}$.
- e) Find the inverse by Gauss-Jordan Elimination :

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}.$$

- f) Find the eigenvalues of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.
- g) Find the eigenvectors of $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.
- h) Prove that $\lim_{x\to 0} \frac{\left(e^{x}-1\right)}{x} = 1$.
- i) If $x = a \left(\sec^2 \theta \right)$ and $y = a \left(\tan^3 \theta \right)$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.







- k) Find Wronskian of x, x^2 , x^3 .
- l) Write the iteration scheme for Runge-Kutta method of order four (RK4) for the initial value problem

$$y' = f(x, y)$$
 with $y(x_0) = y_0$.

GROUP - A

Answer any *three* of the following. $3 \times 12 = 36$

2. a) Show whether following two matrices are commutative or non-commtative :

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}.$$

b) For the matrix $A=\begin{pmatrix}1&2&-4\\2&3&-4\\0&5&1\end{pmatrix}$, verify that $A+A^T$

is a symmetric matrix and $(A - A^T)$ is a skew-symmetric matrix.

- c) In 2-dimensional space, give an example of a set of two vectors, which are linearly independent but not orthogonal. Can you give an example of a set of vectors which are orthogonal to each other but not linearly independent? 3×4
- 3. Diagonalize the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$ by using modal

4. Prove that following *three* vectors arre independent in R^3 .

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$
 , $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$. Use Gram-Schmidt

orthonormalization procedure to find an orthonormal set.

5. Solve the differential equation by using the method of variation of parameters :

$$y^{\parallel} + y^{\parallel} = \sec x$$
.

6. Solve the system $X^{\dagger} = AX$ where $A = \begin{pmatrix} 2 & 8 \\ -1 & -2 \end{pmatrix}$ with initial value $X(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

40972

matrix.

GROUP - B

Answer any *four* of the following.

 $4\times 6=24$

- 7. a) A company sells TV sets with a claim that out of randomly selected 50 TV sets, only 1 may be defective.If 5 sets are selected at random, what is the probability that there would be no defective set?(Use Binomial Distribution).
 - b) In a Board Examination, on an average 3 students get full marks in Mathematics per year. What is the probability that this year 5 students will score full marks in Mathematics? (Use Poisson's distribution)

3 + 3

- 8. It is observed that 20% of the Biotech students go for higher study. What is the probability that out of 300 Biotech students more than 75 go for higher study? (Normal approximation to Binomial distribution)
- 9. Design a decision rule to test the hypothesis that a coin is fair if we take a sample of 100 tosses of the coin at $\alpha=0.05.$

40972

5

[Turn over

- 10. IQs of 14 students from one city showed a mean of 100 and s.d. 10, while IQs of 10 students of another city showed a mean of 110 and s.d. 8. Is there any significant difference at 5% level of significance ? (Use pairwise *t*-test).
- 11. Look at the following survey taken in a city, where the numbers indicated in the table are in the unit of thousand:

	Economic Status		
Time for viewing TV per day	Rich	Middle class	Poor
more than 6 hours	7	5	6
between 3 and 6 hours	9	10	7
less than 3 hours	4	2	5

Decide using χ^2 test at 5% level whether the 'Economic status' and 'time spent for viewing TV' are independent.

12. The points on a rectangular co-ordinate are given as (x, y):

Use linear regression method to find the best fit straight line.

40972

13. Following are yields in tons per hectare for three varieties of wheat *A*, *B* and *C*, grown in four different equivalent plots. Find with *One-way ANOVA* at 5% level whether there is any significant, difference in the yield between the three varieties:

A	48	49	50	49
В	47	49	48	48
C	49	51	50	50

40972 7 [Turn over