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CS/M.TECH/MBIN/SEM-1/MBIN-104/2012-13

2012

MATHEMATICS AND STATISTICS

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *three* questions from **Group-A** and any *four* questions from **Group-B**.

1. Answer any *ten* questions : $10 \times 1 = 10$

a) Show that the following system of equations has infinite number of solutions :

$$x + 2y + 3z = 1, \quad 3x + 4y + 5z = 2, \quad 5x + 6y + 7z = 3.$$

b) If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -3 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 9 & 0 \\ 1 & 1 & 5 \end{pmatrix}$ find matrix C, such that $A + B + C = O$, where O is the null matrix of appropriate order.

c) If $A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$, prove that $A^2 - 3I = 0$, where

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

d) Find the adjoint of $\begin{pmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{pmatrix}$.

e) Find the inverse by Gauss-Jordan Elimination :

$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$.

f) Find the eigenvalues of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

g) Find the eigenvectors of $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

h) Prove that $\lim\limits_{x \to 0} \dfrac{\left(e^x - 1 \right)}{x} = 1$.

i) If $x = a\left(\sec^2 \theta \right)$ and $y = a\left(\tan^3 \theta \right)$, find $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{4}$.

j) Solve the equation $4x\,dy - y\,dx = x^2\,dy$.

k) Find Wronskian of x, x^2, x^3.

l) Write the iteration scheme for Runge-Kutta method of order four (RK4) for the initial value problem

$y' = f(x, y)$ with $y\left(x_0\right) = y_0$.

GROUP – A

Answer any *three* of the following. $3 \times 12 = 36$

2. a) Show whether following two matrices are commutative or non-commtative :

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}.$$

b) For the matrix $A = \begin{pmatrix} 1 & 2 & -4 \\ 2 & 3 & -4 \\ 0 & 5 & 1 \end{pmatrix}$, verify that $A + A^T$ is a symmetric matrix and $\left(A - A^T\right)$ is a skew-symmetric matrix.

c) In 2-dimensional space, give an example of a set of two vectors, which are linearly independent but not orthogonal. Can you give an example of a set of vectors which are orthogonal to each other but not linearly independent ? 3×4

3. Diagonalize the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$ by using modal matrix.

4. Prove that following *three* vectors arre independent in R^3.
$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$. Use Gram-Schmidt orthonormalization procedure to find an orthonormal set.

5. Solve the differential equation by using the method of variation of parameters :

$$y''' + y' = \sec x.$$

6. Solve the system $X' = AX$ where $A = \begin{pmatrix} 2 & 8 \\ -1 & -2 \end{pmatrix}$ with initial value $X(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

GROUP – B

Answer any *four* of the following. $4 \times 6 = 24$

7. a) A company sells TV sets with a claim that out of randomly selected 50 TV sets, only 1 may be defective. If 5 sets are selected at random, what is the probability that there would be no defective set ?

   (Use Binomial Distribution).

   b) In a Board Examination, on an average 3 students get full marks in Mathematics per year. What is the probability that this year 5 students will score full marks in Mathematics ? (Use Poisson's distribution)

   3 + 3

8. It is observed that 20% of the Biotech students go for higher study. What is the probability that out of 300 Biotech students more than 75 go for higher study ? (Normal approximation to Binomial distribution)

9. Design a decision rule to test the hypothesis that a coin is fair if we take a sample of 100 tosses of the coin at $\alpha = 0{\cdot}05$.

10. IQs of 14 students from one city showed a mean of 100 and s.d. 10, while IQs of 10 students of another city showed a mean of 110 and s.d. 8. Is there any significant difference at 5% level of significance ? (Use pairwise *t*-test).

11. Look at the following survey taken in a city, where the numbers indicated in the table are in the unit of thousand :

Time for viewing TV per day	*Economic Status*		
	Rich	Middle class	Poor
more than 6 hours	7	5	6
between 3 and 6 hours	9	10	7
less than 3 hours	4	2	5

Decide using χ^2 test at 5% level whether the 'Economic status' and 'time spent for viewing TV' are independent.

12. The points on a rectangular co-ordinate are given as (x, y) :

(1, 1), (3, 2), (4, 4), (6, 4), (8, 5), (9, 7), (11, 8), (14, 9)

Use linear regression method to find the best fit straight line.

13. Following are yields in tons per hectare for three varieties of wheat *A*, *B* and *C*, grown in four different equivalent plots. Find with *One-way ANOVA* at 5% level whether there is any significant, difference in the yield between the three varieties :

A	48	49	50	49
B	47	49	48	48
C	49	51	50	50