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ADVANCED ENGINEERING MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any four from the rest.

- 1. Choose the correct alternatives for any fourteen of the following: $14\times 1=14$
 - i) The rank of a null matrix of order 2011×2011 is
 - a) 2011

b) 4022

c) 0

- d) none of these.
- ii) 0 is a eigenvalue of a
 - a) singular matrix
- b) non-singular matrix
- c) orthogonal matrix
- d) none of these.
- iii) Every homogeneous system of equations is always consistent.
 - a) True

b) False.

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- The maximum possible rank of the matrix of order iv) 2011×2012 is
 - a) 0

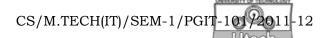
b) 2011

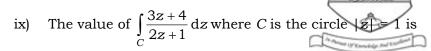
2012 c)

- d) 4023.
- The logical equivalence statement of the following v) compound proposition $\sim (p \lor q)$
 - $\sim (p \wedge q)$ a)
- b) $\sim p \vee q$
- c) $\sim p \wedge \sim q$
- d) none of these.
- Which of the following is not a proposition? vi)
 - Every differential function is continuous a)
 - 2012 > 2011 b)
 - What are you doing? c)
 - d) None of these.
- vii) If f(z) = u(x,y) + iv(x,y) is analytic, then f'(z) =
 - a) $\frac{\partial u}{\partial x} i \frac{\partial v}{\partial x}$ b) $\frac{\partial u}{\partial x} i \frac{\partial u}{\partial y}$
 - c) $\frac{\partial v}{\partial y} i \frac{\partial v}{\partial x}$
- none of these. d)
- viii) If $u(x, y) = 2x x^2 + ky^2$ is harmonic, then k should be
 - 1 a)

2 b)

c) 3 d) 0.





a) $2\pi i$

b) $3\pi i$

c) 4

- d) -4
- x) The function $|z|^2$ is analytic at any point.
 - a) True

b) False.

xi)
$$z = 0$$
 for the function $f(z) = z e^{1/z^2}$ is

- a) Removable singularity b) Pole
- c) Essential singularity d)
 - d) none of these.
- xii) The analytic function whose imaginary part is $e^x \sin y$ is
 - a) e^{iz}

b) e^{-z}

c) e^z

- d) none of these.
- xiii) Value of $\int_C \left(\frac{2}{z} + \frac{3}{z^2}\right) \mathrm{d}z$, C being a path enclosing the origin is
 - a) $6\pi i$

b) 4π*i*

c) $2\pi i$

d) 0.



- xiv) $2\pi\delta \left(w-w_{0}\right)$ is the Fourier transform of
 - a) e^{jw_0t}

b) $\cos w_0 t$

- c) $\sin w_0 t$
- d) none of these.
- xv) Representation of an arbitrary function over the entire interval $(-\infty, \infty)$ is called
 - a) Fourier series
- b) Fourier transform.
- xvi) If a function f is even, all the Fourier coefficients a_n for n = 0, 1, 2, ... are zero; if f is odd, all Fourier coefficients b_n for n = 0, 1, 2, ... are zero.
 - a) True

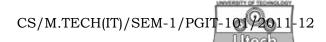
- b) False.
- 2. a) Find the eigenvalues of the following matrix $A = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix} \quad \text{also} \quad \text{find} \quad \text{the eigenvectors}$

corresponding to eigenvalue $\lambda = -2$.

- b) Show that if A is a triangular matrix, the eigenvalues are the diagonal entries.
- c) Define rank of a matrix and find the rank of the following matrix:

$$A = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$$
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3. a) Solve the following system of equations:

$$x + y + z = 10$$

$$2x + 2y + 2z = 20$$

- b) Show that $(p \land q) \rightarrow (p \lor q)$ is a tautology.
- c) What is the truth value of the following statement if the universe of discourse of variable m, n are all integers:

$$\forall n \exists m (n^2 < m)$$

- d) Using truth table show that the following statements are logically equivalent:
 - i) $PV(P \wedge Q)$ and P

ii)
$$P \rightarrow Q$$
 and $\sim P \vee Q$

$$5 + 3 + 3 + 3$$

- 4. a) Show that the sum of all eigenvalues of a square matrix is trace of the matrix.
 - b) Prove that the function f(z) = u + iv where

$$f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}(z \neq 0), f(0) = 0$$

is continuous and that Cauchy-Reimann equations are satisfied at the origin, yet f'(z) does not exist there.

$$7 + 7$$



If f(z) is analytic, prove that 5.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$$

- b) Find the Taylor's and Laurent's series which represent the function $f(z) = \frac{z^2 - 1}{(z + 2)(z + 3)}$ when

 - i) |z| < 2 ii) 2 < |z| < 3
- iii) |z| > 3

7 + 7

- 6. a) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$, if C is the circle |z| = 3.
 - Using the contour integration show that b)

$$\int_{0}^{2\pi} \frac{\mathrm{d}\theta}{5 + 4\cos\theta} = \frac{2\pi}{3} .$$

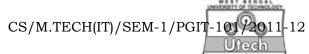
Find the inverse Z-transform of $F(z) = \frac{z^3 - 20z}{(z-2)^3(z-4)}$.

$$5 + 5 + 4$$

- a) $f(x) = \pi^2 x^2$ for $x \in (-\pi, \pi)$. By Fourier series show 7. that $f(x) = \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{-4}{n^2} (-1)^n \cos nx$.
 - b) Mathematically prove that Fourier transform of a function f(t) is given by $F(w) = \int_{-\infty}^{\infty} f(t) e^{-jwt} dt$.

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- 8. a) Explain probability as relative frequency of occurrence.
 - b) What do you mean by mutually exclusive events?
 - c) The set of all possible outcomes of an event is defined by a sample space *S*. Two events *A* and *B* occurring in *S* are related in probability space by the following 3 axioms.
 - i) P(S) = 1
 - ii) $0 \le P(A) \le 1 \& 0 \le P(B) \le 1$
 - iii) Probability of any of A or B occurring $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive.

Explain the above using Venn diagram. Show using set theory how axiom (iii) is to be modified if A and B are not mutually exclusive and hence find probability of A and B jointly occurring for P(A) = 0.2, P(B) = 0.4 and $P(A \cup B) = 0.5$. Also find the probability that none of A or B will occur.

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