Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(IT)/SEM-1/PGIT-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. ***1*** *and any* ***four*** *from the rest.*

1. Choose the correct alternatives for any *fourteen* of the following : $14 \times 1 = 14$

   i) The rank of a null matrix of order 2011×2011 is

      a) 2011 b) 4022

      c) 0 d) none of these.

   ii) 0 is a eigenvalue of a

      a) singular matrix b) non-singular matrix

      c) orthogonal matrix d) none of these.

   iii) Every homogeneous system of equations is always consistent.

      a) True b) False.

iv) The maximum possible rank of the matrix of order 2011×2012 is

a) 0 b) 2011

c) 2012 d) 4023.

v) The logical equivalence statement of the following compound proposition $\sim(p \vee q)$

a) $\sim(p \wedge q)$ b) $\sim p \vee q$

c) $\sim p \wedge \sim q$ d) none of these.

vi) Which of the following is not a proposition ?

a) Every differential function is continuous

b) 2012 > 2011

c) What are you doing ?

d) None of these.

vii) If $f(z) = u(x, y) + iv(x, y)$ is analytic, then $f'(z) =$

a) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$ b) $\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$

c) $\frac{\partial v}{\partial y} - i\frac{\partial v}{\partial x}$ d) none of these.

viii) If $u(x, y) = 2x - x^2 + ky^2$ is harmonic, then k should be

a) 1 b) 2

c) 3 d) 0.

ix) The value of $\int_C \frac{3z+4}{2z+1}\,dz$ where C is the circle $|z| = 1$ is

a) $2\pi i$ b) $3\pi i$

c) 4 d) – 4.

x) The function $|z|^2$ is analytic at any point.

a) True b) False.

xi) $z = 0$ for the function $f(z) = z\,e^{1/z^2}$ is

a) Removable singularity b) Pole

c) Essential singularity d) none of these.

xii) The analytic function whose imaginary part is $e^x \sin y$ is

a) e^{iz} b) e^{-z}

c) e^z d) none of these.

xiii) Value of $\int_C \left(\frac{2}{z} + \frac{3}{z^2}\right) dz$, C being a path enclosing the origin is

a) $6\pi i$ b) $4\pi i$

c) $2\pi i$ d) 0.

xiv) $2\pi\delta(w - w_0)$ is the Fourier transform of

a) $e^{jw_0 t}$ b) $\cos w_0 t$

c) $\sin w_0 t$ d) none of these.

xv) Representation of an arbitrary function over the entire interval $(-\infty, \infty)$ is called

a) Fourier series b) Fourier transform.

xvi) If a function f is even, all the Fourier coefficients a_n for $n = 0, 1, 2, \ldots$ are zero; if f is odd, all Fourier coefficients b_n for $n = 0, 1, 2, \ldots$ are zero.

a) True b) False.

2. a) Find the eigenvalues of the following matrix $A = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$ also find the eigenvectors corresponding to eigenvalue $\lambda = -2$.

b) Show that if A is a triangular matrix, the eigenvalues are the diagonal entries.

c) Define rank of a matrix and find the rank of the following matrix :

$$A = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$$ 6 + 4 + 4

3. a) Solve the following system of equations :

$$x + y + z = 10$$

$$2x + 2y + 2z = 20$$

b) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

c) What is the truth value of the following statement if the universe of discourse of variable m, n are all integers :

$$\forall n \, \exists m \, (n^2 < m)$$

d) Using truth table show that the following statements are logically equivalent :

i) $PV (P \wedge Q)$ and P

ii) $P \rightarrow Q$ and $\sim P \vee Q$ 5 + 3 + 3 + 3

4. a) Show that the sum of all eigenvalues of a square matrix is trace of the matrix.

b) Prove that the function $f (z) = u + iv$ where

$$f (z) = \frac{x^3 (1 + i) - y^3 (1 - i)}{x^2 + y^2} (z \neq 0), f (0) = 0$$

is continuous and that Cauchy-Reimann equations are satisfied at the origin, yet $f' (z)$ does not exist there.

7 + 7

5. a) If $f(z)$ is analytic, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$$

b) Find the Taylor's and Laurent's series which represent the function $f(z)=\dfrac{z^2-1}{(z+2)(z+3)}$ when

i) $|z|<2$ ii) $2<|z|<3$ iii) $|z|>3$

7 + 7

6. a) Evaluate $\int_C \dfrac{e^{2z}}{(z+1)^4}\,dz$, if C is the circle $|z| = 3$.

b) Using the contour integration show that

$$\int_0^{2\pi}\frac{d\theta}{5+4\cos\theta}=\frac{2\pi}{3}.$$

c) Find the inverse Z-transform of $F(z)=\dfrac{z^3-20z}{(z-2)^3(z-4)}$.

5 + 5 + 4

7. a) $f(x)=\pi^2-x^2$ for x € $(-\pi,\pi)$. By Fourier series show that $f(x)=\dfrac{2\pi^2}{3}+\sum_{n=1}^{\infty}\dfrac{-4}{n^2}(-1)^n\cos nx$.

b) Mathematically prove that Fourier transform of a function $f(t)$ is given by $F(w)=\int_{-\infty}^{\infty} f(t)\,e^{-jwt}dt$.

7 + 7

8. a) Explain probability as relative frequency of occurrence.

   b) What do you mean by mutually exclusive events ?

   c) The set of all possible outcomes of an event is defined by a sample space S. Two events A and B occurring in S are related in probability space by the following 3 axioms.

   i) $P(S) = 1$

   ii) $0 \leq P(A) \leq 1$ & $0 \leq P(B) \leq 1$

   iii) Probability of any of A or B occurring $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive.

   Explain the above using Venn diagram. Show using set theory how axiom (iii) is to be modified if A and B are not mutually exclusive and hence find probability of A and B jointly occurring for $P(A) = 0{\cdot}2$, $P(B) = 0{\cdot}4$ and $P(A \cup B) = 0{\cdot}5$. Also find the probability that none of A or B will occur. 4 + 2 + 8

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