



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.Tech(EE)/SEM-1/MEE-1.5.4(BL)/2010-11  
2010-11**

**OPTIMAL CONTROL & ESTIMATION**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

Answer any *five* of the following.  $5 \times 14 = 70$

1. a) Derive Euler-Lagrange equation to determine a curve  $x(t)$  connecting two points  $(x_0, t_0)$  and  $(x_r, t_r)$  such that the integral along the curve of some given function  $F(x, \dot{x}, t)$  is a minimum. 7

- b) Find the external curves for the functional

$$J = \int_0^1 \{ 1 + \dot{x}^2 e(t) \} dt. \quad 7$$

2. a) State Pontryagin's maximum principle. Discuss the steps involved in solving optimal control problems using this principle. 7



- b) The dynamics of a system is described by,

$$\dot{X}(t) = x^2(t)$$

$$X^2(t) = u(t)$$

This system is to be controlled, minimizing the PI,

$$\int_0^1 f^2$$

$$J(X, u) = \int_0^1 u^2(t) dt$$

Find a set of necessary conditions for the optimal control. 7

3. a) Using the definition, determine the differential of the functional,

$$J(X) = \int_0^1 [x(t) + x(t)X^2(t) + x(t) + 2x(t)X^2(t)] dt$$

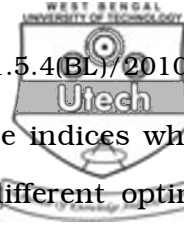
Assume that the end points are specified. 3

- b) Find the extremals for the functional,

$$J(X) = \int_0^1 [x(t)^2 + 2x(t)X^2(t) + X^2(t)] dt$$

with  $x(0) = 0, x(1) = 1, X(0) = 0, x(1) = 1$ . 3

- c) Let  $f(x) = -xtX^2$  and  $g(x) = xt^2 + x - 1$ . What are the potential candidates for minima of  $f(x)$  subject to the condition  $g(x) = 0$ ? 8



4. a) Briefly discuss the different performance indices which are generally used for formulation of different optimal control problems. 7

- b) The auto-correlation function of a noise signal is given by,

$R_x(t) = \frac{J}{2} \exp(-\rho |t|)$  where  $\rho$  and  $J$  are two constants.

Obtain the transfer function of the shaping filter that will convert a unity white noise signal to this particular noise signal. 7

5. a) Distinguish between a random variable and a random process. What do you mean by strict-sense stationarity, wide-sense stationarity and ergodicity of a random process ? 6

- b) Show how the mean and covariance of the state vector propagate through time when a linear dynamic system is excited by a zero-mean white noise input. 8

6. a) What do you mean by prediction, filtering and smoothing problems ? In what sense is the Kalman filter an optimal filter ? 4



- b) A linear discrete-time system is described by,

$$x_k = X x_{k-1} + W u_{k-1}$$

$$Z_k = X x_k + V u_k$$

where the process noise and the measurement noise are zero-mean white noises with intensities 1 and 2 respectively.

Calculate the Kalman gain  $K_k$  and the estimation error covariance  $P_k$  for  $k = 1$  and 2 assuming  $P_0 = 1$ . Also determine their steady state values. 6

- c) Explain how coloured process disturbance situation can be accommodated in a standard Kalman filter setting. 4
7. a) Derive the return difference inequality property of the infinite-time LQR controller and show that this controller yields a minimum gain margin of infinity and phase margin of  $60^\circ$  for a SISO minimum phase system. 9
- b) Write a short note on Loop Transfer Recovery method. 5

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