Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(EE)/SEM-1/MEE-1.5.4(BL)/2010-11

2010-11

OPTIMAL CONTROL & ESTIMATION

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* of the following. $5 \times 14 = 70$

1. a) Derive Euler-Lagrange equation to determine a curve $x(t)$ connecting two points (xo, to) and (xr, tr) such that the integral along the curve of some given function $F(x, x, t)$ is a minimum. 7

   b) Find the external curves for the functional

$$J = if \left\{ 1 + \therefore e(t) \right\} dt. \quad 7$$

2. a) State Pontryagin's maximum principle. Discuss the steps involved in solving optimal control problems using this principle. 7

b) The dynamics of a system is described by,

$Xt (t) = x\ 2 (t)$

$X2 (t) = u (t)$

This system is to be controlled, minimizing the PI,

$1 f 2$

$J (X, u) = "2 .b\ u\ 2 (t)$ dt

Find a set of necessary conditons for the optimal control. 7

3. a) Using the definition, determine the differential of the functional,

$J (X) = f : [x \sim (t) + x) (t) X2 (t) + x \sim (t) + 2x) (t) X2 (t) \} it$

Assume that the end points are specified. 3

b) Find the extremals for the functional,

$J (X) = r/2 [x) 2 (t) + 2x) (t) X2 (t) + X \sim (t) \} it$

with $x) (O) = O, x) (nl\ 2) = 1, X2 (O) = O, x\ 2 (nl2) = 1.$ 3

c) Let $f (x) = - xt\ X2$ and $g (x) = xt2 + x1 - 1$. What are the potential candidates for minima of $f (x)$ subject to the condition $g (x) = 0$? 8

4. a) Briefly discuss the different performance indices which are generally used for formulation of different optimal control problems. 7

b) The auto-correlation function of a noise signal is given by,

Rx (*T*) = (*J*2 exp (– pITI) where (7 and *fJ*) are two constants.

Obtain the transfer function of the shaping filter that will convert a unity white noise signal to this particular noise signal. 7

5. a) Distinguish between a random variable and a random process. What do you mean by strict-sense stationarity, wide-sense stationarity and ergodicity of a random process ? 6

b) Show how the mean and covariance of the state vector propagate through time when a linear dynamic system is excited by a zero-mean white noise input. 8

6. a) What do you mean by prediction, filtering and smoothing problems ? In what sense is the Kalman filter an optimal filter ? 4

b) A linear discrete-time system is described by,

$$x\ k = X\ k - I + W\ k - I.$$

$$Z\ k = X\ k + V\ k$$

where the process noise and the measurement noise are zero-mean white noises with intensities 1 and 2 respectively.

Calculate the Kalman gain Kk and the estimation error covariance Pk for k = 1 and 2 assuming P_o =1 O. Also determine their steady state values. 6

c) Eplain how coloured process disturbance situation can be accommodated in a standard Kalman filter setting. 4

7. a) Derive the return difference inequality property of the infinite-time LQR controller and show that this controller yields a minimum gain margin of infinity and phase margin of 60° for a SISO minimum phase system. 9

b) Write a short note on Loop Transfer Recovery method. 5