



Name :

Roll No. :

Invigilator's Signature :

**CS/M.Tech(EE)/SEM-1/CI-1.2/2009-10
2009**

LINEAR CONTROL THEORY

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer questions 1 and any *four* from the rest.

$$5 \propto 14 = 70$$

1. Choose the correct alternatives for the following : $7 \propto 2 = 14$

- i) Feedback control systems are
 - a) insensitive to both forward and feedback path parameter changes
 - b) less sensitive to feedback path parameter changes than to forward path parameter changes
 - c) less sensitive to forward path parameter changes than to feedback path parameter changes
 - d) equally sensitive of forward and feedback path parameter changes.



- ii) The number of forward paths and the sum of gain products of two non-touching loops in the signal flow graph of Figure 1 are

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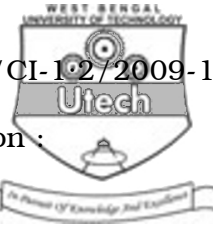
Figure 1

- a) $3, t_{23} \cdot t_{32} \cdot t_{44}$
- b) $2, t_{23} \cdot t_{32} + t_{34} \cdot t_{43}$
- c) $1, t_{23} \cdot t_{32} + t_{34} \cdot t_{43} + t_{44}$
- d) $3, t_{24} \cdot t_{43} \cdot t_{32} + t_{44} \cdot$
- iii) The unit-impulse response of a unity feedback system is given by :

$$c(t) = -t.e^{-t} + 2e^{-t}, (t \geq 0)$$

The open-loop transfer function is equal to

- a) $\frac{2S + 1}{(S + 1)^2}$
- b) $\frac{2S + 1}{S^2}$
- c) $\frac{S + 2}{(S + 1)^2}$
- d) $\frac{S + 1}{S^2}$



- iv) A system is described by the state-equation :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

The state-transition matrix of the system is

- a) $\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}$ b) $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$
 c) $\begin{bmatrix} e^{2t} & 1 \\ 1 & e^{2t} \end{bmatrix}$ d) $\begin{bmatrix} e^{-2t} & 1 \\ 1 & e^{-2t} \end{bmatrix}$.

- v) The polar plot (for positive frequencies) for the open-loop transfer function of a unity feedback control system is shown in Figure 2. The phase margin and gain margin of the system are

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Figure 2

- a) 150° and 4 b) 150° and $\frac{3}{4}$
 c) 30° and 4 d) 30° and $\frac{3}{4}$.



- vi) A phase-lead compensator has the transfer function

$$G_c(s) = \frac{0.5S + 1}{0.05S + 1}$$

The maximum phase-angle lead provided by this compensator is

- a) 52 deg at 4 rad/s b) 52 deg at 10 rad/s
c) 55 deg at 12 rad/s d) None of these.
- vii) A particular control system is described by the following state-equation :

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \text{ and } y = \begin{bmatrix} 2 & 0 \end{bmatrix} \underline{x}$$

The transfer function $\frac{Y(s)}{U(s)}$ of the system is

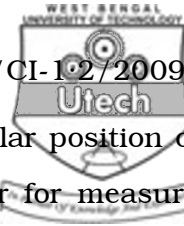
- a) $\frac{1}{2S^2 + 3S + 1}$ b) $\frac{2}{2S^2 + 3S + 1}$
c) $\frac{1}{S^2 + 3S + 2}$ d) $\frac{2}{S^2 + 3S + 2}$.
2. A lead compensator is to be designed for a unity feedback system having an open-loop transfer function

$$G(S) = \frac{K}{S(S + 2.5)}$$

to meet the following performance specifications

- i) phase margin = 55°
ii) velocity error constant $K_v \geq 12 \text{ sec}^{-1}$

Determine the required value of gain K and choose suitable values for the compensator parameters using Bode plots.



3. a) Show a scheme for controlling the angular position of a shaft using potentiometer error detector for measuring any deviation of output shaft *w.r.t.* reference position. The amplified error signal is to be fed to an armature controlled *d.c.* motor whose shaft is coupled to load shaft through a gear-train.

Develop the block diagram and derive the over-all transfer function. Compute the undamped natural frequency and damping ratio of the closed-loop system if the system parameters are :

$$K_p = \text{pot.sensitivity} = 2\text{V/rad}, K_A = 10\text{V/V}, \\ R_a = 0.2 \Omega, L_a \text{ negligible. } K_T = 12 \times 10^{-5} \text{ Nm/A}, \\ K_b = 12 \times 10^{-5} \text{ V/rad/s}, \text{ Gear ratio is } 100 : 10.$$

Equivalent moment of inertia and co-efficient of viscous friction referred to motor side are

$$15 \times 10^{-5} \text{ kg-m}^2 \text{ and } 10 \times 10^{-5} \text{ Nm/rad/s} \\ \text{respectively.} \quad 12$$

- b) Obtain the eigenvectors of the matrix $A = \begin{bmatrix} -1 & -1 \\ 2 & -4 \end{bmatrix}$.

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4. a) Obtain the solution of the state equation :

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\underline{x}(0) = [0 \ 1]^T \text{ and } u \text{ is a unit step input.} \quad 7$$



- b) A position control system with velocity feedback damping is shown in Figure 3.

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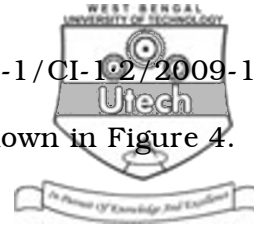
Figure 3

- i) Determine the settling time and maximum overshoot in response to a unit step input for $\alpha = 2$. What is the steady-state error to a unit ramp input ?
 - ii) Find the value of α so that the closed-loop damping ratio becomes 0.6. 7
5. a) Derive a linearised state-model of a time-invariant system represented by the second order non-linear differential equation :

$$\frac{d^2x}{dt^2} + x^2 \left(\frac{dx}{dt} - 2 \right) + x = 0.$$

Determine the points of equilibrium. Investigate the stability of the system near each point of equilibrium.

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- b) Derive a state-model for the circuit shown in Figure 4.

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Figure 4

6. a) A linear system is described by :

$$\dot{\underline{x}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{-5}{6} & \frac{-13}{6} & \frac{-1}{3} \end{bmatrix} \underline{x} + \begin{bmatrix} 3 & 1 \\ 2 & 0 \\ -1 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix} \underline{x}$$

Determine whether the system is completely controllable.

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- b) The state-model of a third order control system is described by :

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u ; y = [1 \ 0 \ 0] \underline{x}$$

It is desired to modify both transient and steady-state performance of the system employing state-variable feedback. To meet the design specifications, the dominant poles in the closed loop are to be located at $S_d = -1 \pm j1$ and the third pole has to be shifted to $S = -6$. The system should be capable to track a step input command with zero steady-state errors. Compute the required controller gain R and the feedback vector K^T . Also show an implementation scheme assuming all state-variables are available for measurement.

7. a) A unity feedback control system has an open-loop transfer function given by

$$G(s)H(s) = \frac{10}{s(s+1)(s+4)}$$

Draw the complete Nyquist plot and investigate the closed-loop system stability. 12

- b) Determine the position and velocity error constants for the system considered in (a). 2

8. Write notes on any *three* of the following :

- Ziegler-Nicholas Rules for tuning PID controllers.
- Speed regulator employing armature controlled d.c. servo-motor and its closed-loop time-response characteristic to a step change in set-point value.
- Homogeneous solution and forced solution of linear time-invariant state equation and state transition matrix (STM).
- The locus of the roots of characteristic equation of a second order linear system when undamped natural frequency is held constant and damping ratio is varied from 0 to α and the classification of system dynamics w.r.t. values of φ .