Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / M.Tech(EE) / SEM-1 / CI-1·2 / 2009-10

2009

LINEAR CONTROL THEORY

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer questions 1 and any *four* from the rest.

5 ∞ 14 = 70

1. Choose the correct alternatives for the following :   7 ∞ 2 = 14

   i) Feedback control systems are

      a) insensitive to both forward and feedback path parameter changes

      b) less sensitive to feedback path parameter changes than to forward path parameter changes

      c) less sensitive to forward path parameter changes than to feedback path parameter changes

      d) equally sensitive of forward and feedback path parameter changes.

ii) The number of forward paths and the sum of gain products of two non-touching loops in the singal flow graph of Figure 1 are

Dia

Figure 1

a) 3, $t_{23} . t_{32} . t_{44}$

b) 2, $t_{23} . t_{32} + t_{34} . t_{43}$

c) 1, $t_{23} . t_{32} + t_{34} . t_{43} + t_{44}$

d) 3, $t_{24} . t_{43} . t_{32} + t_{44}$.

iii) The unit-impulse response of a unity feedback system is given by :

$$c(t) = -t.e^{-t} + 2e^{-t}, \ (t \geq 0)$$

The open-loop transfer function is equal to

a) $\dfrac{2S+1}{(S+1)^2}$ b) $\dfrac{2S+1}{S^2}$

c) $\dfrac{S+2}{(S+1)^2}$ d) $\dfrac{S+1}{S^2}$.

iv) A system si decribed by the state-equation :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

The state-transition matrix of the system is

a) $\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}$ b) $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$

c) $\begin{bmatrix} e^{2t} & 1 \\ 1 & e^{2t} \end{bmatrix}$ d) $\begin{bmatrix} e^{-2t} & 1 \\ 1 & e^{-2t} \end{bmatrix}$.

v) The polar plot (for positive frequencies) for the open-loop transfer function of a unity feedback control system is shown is Figure 2. The phase margin and gain margin of the system are

Dia.

Figure 2

a) 150° and 4 b) 150° and $\frac{3}{4}$

c) 30° and 4 d) 30° and $\frac{3}{4}$.

vi) A phase-lead compensator has the transfer function

$$G_c(s) = \frac{0{\cdot}5S + 1}{0{\cdot}05S + 1}$$

The maximum phase-angle lead provided by this compensator is

a) 52 deg at 4 rad/s b) 52 deg at 10 rad/s

c) 55 deg at 12 rad/s d) None of these.

vii) A particular control system is described by the following state-equation :

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \text{ and } y = [\,2 \quad 0\,]\, \underline{x}$$

The transfer function $\frac{Y(s)}{U(s)}$ of the system is

a) $\frac{1}{2S^2 + 3S + 1}$ b) $\frac{2}{2S^2 + 3S + 1}$

c) $\frac{1}{S^2 + 3S + 2}$ d) $\frac{2}{S^2 + 3S + 2}$.

2. A lead compensator is to be designed for a unity feedback system having an open-loop transfer function

$$G(S) = \frac{K}{S(S + 2{\cdot}5)}$$

to meet the following performance specifications

i) phase margin = 55°

ii) velocity error constant $K_v \geq 12\ \text{sec}^{-1}$

Determine the required value of gain K and choose suitable values for the compensator parameters using Bode plots.

3. a) Show a scheme for controlling the angular position of a shaft using potentiometer error detector for measuring any deviation of output shaft *w.r.t.* reference position. The amplified error signal is to be fed to an armature controlled *d.c.* motor whose shaft is coupled to load shaft through a gear-train.

Develope the block digram and derive the over-all transfer function. Compute the undamped natural frequency and damping ratio of the closed-loop system if the system parameters are :

K_P = pot.sensitivity = 2V/rad, K_A = 10V/V,
R_a = 0·2 Ω, L_a negligible. $K_T = 12 \infty 10^{-5}$ nm/A,
$K_b = 12 \infty 10^{-5}$ V/rad/s, Gear ratio is 100 : 10.

Equivalent movment of inertia and co-efficient of viscous friction referred to motor side are $15 \infty 10^{-5}$ kg-m^2 and $10 \infty 10^{-5}$ nm/rad/s respectively. 12

b) Obtain the eigenvectors of the matrix $A = \begin{bmatrix} -1 & -1 \\ 2 & -4 \end{bmatrix}$. 2

4. a) Obtain the solution of the state equation :

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$\underline{x}(0) = [\,0\ 1\,]^T$ and u is a unit step input. 7

b) A position control system with velocity feedback damping is shown in Figure 3.

Dia.

Figure 3

i) Determine the settling time and maximum overshoot in response to a unit step input for $\alpha = 2$. What is the steady-state error to a unit ramp input ?

ii) Find the value of α so that the closed-loop damping ratio becomes 0·6. 7

5. a) Derive a linearised state-model of a time-invariont system represented by the second order non-linear differential equation :

$$\frac{d^2x}{dt^2} + x^2\left(\frac{dx}{dt} - 2\right) + x = 0.$$

Determine the points of equilibrium. Investigate the stability of the system near each point of equilibrium. 8

b) Derive a state-model for the circuit shown in Figure 4. 6

Dia.

Figure 4

6. a) A linear system is described by :

$$\dot{\underline{x}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{-5}{6} & \frac{-13}{6} & \frac{-1}{3} \end{bmatrix} \underline{x} + \begin{bmatrix} 3 & 1 \\ 2 & 0 \\ -1 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix} \underline{x}$$

Determine whether the system is competely controllable. 3

b) The state-model of a third order control system is described by :

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u \; ; \; y = [\; 1 \;\; 0 \;\; 0 \;] \; \underline{x}$$

It is desired to modify both transient and steady-state performance of the system employing state-variable feedback. To meet the design specifications, the dominant poles in the closed loop are to be located at $S_d = -1 \pm j\,1$ and the third pole has to be shifted to $S = -6$. The system should be capable to track a step input command with zero steady-state errors. Compute the required controller gain R and the feedback vector K^T . Also show an implementation scheme assuming all state-variables are available for measurement.

7. a) A unity feedback control system has an open-loop transfer function given by

$$G(s)H(s) = \frac{10}{s(s+1)(s+4)}$$

Draw the complete Nyquist plot and investigate the closed-loop system stability. 12

b) Determine the position and velocity error constants for the system considered in (a). 2

8. Write notes on any *three* of the following :

a) Ziegler-Nicholas Rules for tuning PID controllers.

b) Speed regulator employing armature controlled *d.c.* servo-motor and its closed-loop time-response characteristic to a step change in set-point value.

c) Homogeneous solution and forced solution of linear time-invariant state equation and state transition matrix (STM).

d) The locus of the roots of characteristic equation of a second order linear system when undamped natural frequency is held constant and damping ratio is varied from 0 to α and the classification of system dynamics *w.r.t.* values of φ.