Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(EE)/SEM-1/EMM-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions   5 × 14 = 70

1. a) Determine and graph the image of $| z - a | = a$ under the transformation $w = z^2$. 5 + 2

   b) Show that the transformation $w = \dfrac{5 - 4z}{4z - 2}$ transform the circle $| z | = 1$ into a circle of radius unity in the w-plane and find he centre of the circle. 6 + 1

2. a) Evaluate $\int_C | z |^2 \, dz$ where c is the square with vartices at (0, 0), (1, 0), (1, 1), (0, 1). 4

   b) Evaluate $\oint_C \dfrac{1}{z^2 + 9} \, dz$ where c is $| z - 3i | = 4$. 5

   c) Prove that $\dfrac{1}{2\pi i} \int_C \dfrac{e^{zt}}{z^2 + 1} \, dz = \sin t$,

   where $t > 0$ and c is the circle $| z | = \pi$. 5

3. a) Solve the following difference equation : 7

$y_{n+2} - 5y_{n+1} + 6y_n = n + 2^n$.

b) Form the divided difference table of the following tabular values of a function $f(x)$ and find the interpolating polynomial.

x	-2	-1	0	1	2	3
$f(x)$	70	11	4	1	2	55

7

4. a) Solve the following system of equations, correct to 3 decimal places, by Newton-Rephson method with (0, 0) as initial app[roximation : 7

$x^3 + y^3 - 6x + 3 = 0$

$x^3 - y^3 - 6y + 2 = 0.$

b) By using LU decomposition method, solve the following system : 7

$2x + y + z = 3$

$x + 3x + z = -2$

$x + y + 4z = -6.$

5. a) Use Adams' predictor-corrector method to obtain the solution of the equation

$$\frac{dy}{dx} = x^2 + y^2 \text{ at } x = 1{\cdot}4,$$

given that $y(1) = 0$, $y(1{\cdot}1) = 0{\cdot}11072$, $y(1{\cdot}2) = 0{\cdot}24631$, $y(1{\cdot}3) = 0{\cdot}41357$. 7

b) Using Bender-Schmidt method find the values of $u(x, t)$ satisfying the parabolic equation

$u_t = 4\,u_{xx}$

subject to

boundary conditions $u(0, t) = 0 = u(8, t)$

and initial condition $(x, 0) = 4x - \frac{1}{2}x^2$

at the points $x = i$ where $i = 0, 1, 2, \ldots, 7$ and $t = \frac{j}{8}$,

$j = 0, 1, 2, \ldots\ 5$. 7

6. a) Find the curve c passing through two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ such that the rotation of the curve c about x-axis generates a surface of revolution having minimum surface area. 7

b) Find the extremal of the following functional :

$$\int_0^{\pi/2} (y'^2 - y^2 + 2xy)\,dy;\ y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 0. \qquad 7$$

7. a) Use Bellman's principle of optimality to minimize $z = y_1 + y_2 + \ldots + y_n$, subject to the constraint:

$y_1 \cdot y_2 \cdot \ldots\, y_n = d.$ 7

b) Show that the system of equations :

$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$
$$x - y + z = -1$$

is consistent and solve it. 7

8. a) Let V and W be vector spaces over a field F and V is finite dimensional. If $T : V \rightarrow W$ be a linear mapping, then prove that

rank of T + nullity of T = dim V. 7

b) Let $\{ \alpha_1 = (1, 1, 1), \alpha_2 = (0, 1, 1), \alpha_3 = (0, 0, 1) \}$ be a basis of the Euclidean space $V_3(R)$. Use Gram-Schmidt process of orthogonalisation to obtain an orthonormal basis from $\{ \alpha_i \}$. 7