Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(EE)/SEM-1/EAM-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *seven* questions : $7 \times 2 = 14$

i) Check the nature of the singularity of

$$f(z) = ze^{\frac{1}{z}} \text{ at } z = \infty .$$

ii) Determine the critical points of the bilinear transformation $w = \dfrac{a + bz}{c + dz}$.

iii) Find the value of $\sqrt[3]{25}$ using Newton-Raphson method.

iv) Find $y(3)$ from the following table :

x	0	2	4	6
y	1	7	21	43

v) Solve the differential equation $\dfrac{dy}{dx} = x + y, \ y(0) = 1$ at $x = 1.2$ by modified Euler's method.

vi) Show that $\lim\limits_{(x, y) \to (0, 0)} \dfrac{x^2 y}{x^4 y^2}$ does not exist.

vii) Show that in a vector space V over F

$$(a - b) v = av - bv; \; a, b \in F, \; v \in V.$$

viii) Prove that the set of vectors $\{ (1, 2, 3), (2, 1, 3), (0, 0, 0) \}$ is linearly dependent.

ix) Find the rank of the matrix $\begin{pmatrix} 2 & 2 \\ -3 & -3 \end{pmatrix}$ using definition.

2. a) Show that $f (z) = |z|^2$ is differentiable at the point $z = 0$, but not analytic there. 4

b) Prove that $f (z) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & z \neq 0 \\ 0 & , z = 0 \end{cases}$ is discontinuous at $z = 0$. 5

c) Prove that $u (x, y) = \frac{1}{2} \log \left(x^2 + y^2 \right)$ is harmonic and find its conjugate harmonic function $v (x, y)$ so that $f = u + iv$ is analytic. 5

3. a) Solve the differential equation $\dfrac{dy}{dx} = 2x + y + 1$, $y (0) = 1$ at $x = 0.2$ by 4th order Runge-Kutta method. 4

b) Solve the following equations by Gauss-Seidel method (Three steps only) :

$$x + 2y + 10z = 35$$
$$2x + 10y + z = -15$$
$$2x + y + 10z = 34.$$
5

c) Derive the recurrence formula to solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ under bounded boundary condition. 5

4. a) Use dynamic programming to solve the following problem :

$$\text{Maximize } Z = y_1, y_2, y_3$$

subject to the constraint

$$y_1 + y_2 + y_3 = 10$$

and $y_1, y_2, y_3 \geq 0$. 7

b) Sate the Kuhn-Tucker necessary and sufficient conditions in non-linear programming. 3

c) Solve the following problem :

$$\text{Maximize } Z = 10x_1 - x_1^2 + 10x_2 - x_2^2$$

subject to the constraints

$$x_1 + x_2 \leq 9$$

$$x_1 - x_2 \leq 6$$

and $x_1, x_2 \geq 0$. 4

5. a) Is the vector (2, – 5, 3) in the subspace R^3 spanned by the vectors (1, – 3, 2), (2, – 4 – 1) and (1, – 5, 7) ? 5

b) Let $\phi : R^3 \to R^3$ is defined by

$$\phi\left(x_1, x_2, x_3 \right) = \left(x_1 + 1, x_2 + 1, x_3 + 1 \right); \left(x_1, x_2, x_3 \right) \in R^3,$$

then show that ϕ is not a linear transformation. 5

c) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 2 & 1 & -1 & 0 \end{pmatrix}$. 4

6. a) Find the Laurent's series for $f(z) = \dfrac{1}{(z+1)(z+3)}$ valid in : (a) $1 < |z| < 3$ (b) $0 < |z+1| < 2$. 4

b) For what value of k, the following equations $x + y + z = 1$, $2x + y + 4z = k$, $4x + y + 10z = k^2$ have a solution and solve them completely in each case. 5

c) Find the eigenvalues and the eigenvectors for greatest eigenvalue of the matrix :

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$ 5

7. a) Solve $x^3 + 3x - 10 = 0$ by the method of Regula-Falsi correct up to two decimal places. 4

b) Find the value of $y(4)$ from the following table : 3

x	0	3	5	6
y	2	10	25	30

c) Show that the necessary condition for

$$I = \int_{x_1}^{x_2} f(x, y, y')\, dx$$ to be an extemum is that

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0.$$ 7