Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(EE)/SEM-1/MTM-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. 5 × 14 = 70

1. a) Prove Cauchy's integral formula for multiply connected regions.

   b) Expand $f(z) = \dfrac{3z-1}{z^2-2z-3}$ in Laurent series valid for $1 < |z| < 3$.

   c) Evaluate the integral $\oint_C \dfrac{z}{(16-z^2)(z+i)}\,dz$ where C is the circle : $|z-4| = 2$. 6 + 4 + 4

2. a) Define Bellman principle of optimality.

   Use dynamic programming to solve the following problem :

   Minimize $z = y_1^2 + y_2^2 + y_3^2$

   Subject to the constraints : $y_1 + y_2 + y_3 \geq 15$ and $y_1, y_2, y_3 \geq 0$.

b) Using Contour integration evaluate

$$\int_0^{2\pi} \frac{d\theta}{13 + 5 \sin \theta}.$$

8 + 6

3. a) Use Runge-Kutta method to compute $y(0.4)$ from $\frac{dy}{dx} = x + y$ with $y(0) = 1$, Taking $h = 0.1$.

b) Find the greatest eigenvalue and the corresponding eigenvector of the matrix

$$\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

c) Solve the system of equations by Gauss-Jordon's matrix inversion method.

$$3x + 4y + 2z = 15$$

$$5x + 2y + z = 18$$

$$2x + 3y + 2z = 10$$

6 + 4 + 4

4. a) What is a non-linear programming problem ? State Kuhn-Tucker necessary and sufficient conditions in non-linear programming.

b) Solve the non-linear programming problem :

Optimize $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1 x_2$

Subject to the constraints :

$x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20.$

7 + 7

5. a) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 6$.

b) State and prove Cauchy's Residues theorem of complex variables.

c) Find the general bilinear transformation which maps the unit circle $|z| = 1$ onto $|w| = 1$ and the points $z = 1$ to $w = 1$ and $z = -1$ to $w = -1$. 5 + 6 + 3

6. a) Find the dimension of subspaces of R^3 defined by

$$S = \{ (x, y, z) : 2x + y - z = 0 \}.$$

b) Show that the mapping $T : R^2 \rightarrow R^3$ defined as $T(a, b) = (a + b, a - b, b)$ is a linear transformation from R^2 to R^3. Find $KerT$ and ImT.

Also verify that $dimKerT + dimImT = 2$. 6 + 8

7. a) Determine the condition for which the system

$$x + y + z = 1$$

$$x + 2y - z = b$$

$$5x + 7y + az = b^2.$$

Admits of (i) only one solution (ii) no solution (iii) many solutions.

b) Polynomial $P_3(x) = x^3 - x^2 + 2x - 2$ interpolates $f(x)$ at the points $x = -2, 0, 1$ and 2. Now one more data $f(4) = -90$ is added to get the interpolating polynomial $P_4(x) = P_3(x) + g(x)$. Find $g(x)$ and hence interpolate $f(3)$. 6 + 8