

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.TECH(EE)/SEM-1/MTM-101/2012-13**

**2012**

**ADVANCED ENGINEERING MATHEMATICS**

*Time Allotted : 3 Hours*

*Full Marks : 70*

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

Answer any five questions.

5 × 14 = 70

1. a) Prove Cauchy's integral formula for multiply connected regions.

- b) Expand  $f(z) = \frac{3z - 1}{z^2 - 2z - 3}$  in Laurent series valid for  $1 < |z| < 3$ .

- c) Evaluate the integral  $\oint_C \frac{z}{(16 - z^2)(z + i)} dz$  where  $C$  is the circle  $|z - 4| = 2$ . 6 + 4 + 4

2. a) Define Bellman principle of optimality.

Use dynamic programming to solve the following problem :

$$\text{Minimize } z = y_1^2 + y_2^2 + y_3^2$$

Subject to the constraints :  $y_1 + y_2 + y_3 \geq 15$  and  $y_1, y_2, y_3 \geq 0$ .



- b) Using Contour integration evaluate

$$\int_0^{2\pi} \frac{d\theta}{13 + 5 \sin \theta}.$$

8 + 6

3. a) Use Runge-Kutta method to compute  $y(0.4)$  from  $\frac{dy}{dx} = x + y$  with  $y(0) = 1$ , Taking  $h = 0.1$ .

- b) Find the greatest eigenvalue and the corresponding eigenvector of the matrix

$$\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

- c) Solve the system of equations by Gauss-Jordan's matrix inversion method.

$$3x + 4y + 2z = 15$$

$$5x + 2y + z = 18$$

$$2x + 3y + 2z = 10$$

6 + 4 + 4

4. a) What is a non-linear programming problem ? State Kuhn-Tucker necessary and sufficient conditions in non-linear programming.

- b) Solve the non-linear programming problem :

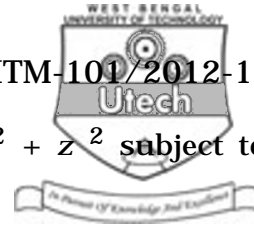
$$\text{Optimize } Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

Subject to the constraints :

$$x_1 + x_2 + x_3 = 15$$

$$2x_1 - x_2 + 2x_3 = 20.$$

7 + 7



5. a) Find the minimum value of  $x^2 + y^2 + z^2$  subject to the condition  $x + y + z = 6$ .  
 b) State and prove Cauchy's Residues theorem of complex variables.  
 c) Find the general bilinear transformation which maps the unit circle  $|z| = 1$  onto  $|w| = 1$  and the points  $z = 1$  to  $w = 1$  and  $z = -1$  to  $w = -1$ . 5 + 6 + 3

6. a) Find the dimension of subspaces of  $R^3$  defined by  

$$S = \{ (x, y, z) : 2x + y - z = 0 \}.$$
 b) Show that the mapping  $T : R^2 \rightarrow R^3$  defined as  $T(a, b) = (a + b, a - b, b)$  is a linear transformation from  $R^2$  to  $R^3$ . Find  $\text{Ker}T$  and  $\text{Im}T$ .

Also verify that  $\dim \text{Ker}T + \dim \text{Im}T = 2$ . 6 + 8

7. a) Determine the condition for which the system

$$x + y + z = 1$$

$$x + 2y - z = b$$

$$5x + 7y + az = b^2.$$

Admits of (i) only one solution (ii) no solution (iii) many solutions.

- b) Polynomial  $P_3(x) = x^3 - x^2 + 2x - 2$  interpolates  $f(x)$  at the points  $x = -2, 0, 1$  and  $2$ . Now one more data  $f(4) = -90$  is added to get the interpolating polynomial  $P_4(x) = P_3(x) + g(x)$ . Find  $g(x)$  and hence interpolate  $f(3)$ . 6 + 8

