Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(EE)/SEM-1/MMA-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. a) Write down fourth order Runge-Kutta formula for solving the differential equation

$$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0.$$ 3

b) Determine the maximum and minimum values of the function $f(x) = 12x^5 - 45x^4 + 40x^3 + 5$. 3

c) State the necessary and sufficient condition for optimization of multivariable objective function with equality consants. 2

d) Evaluate $\oint_C \frac{2z}{z-4}\,dz$, where $C : |z-1| = 2$. 2

e) Show that the vectors (1, 1, 0), (1, 1, 1) and (0, 1, – 1) are linearly independent. 2

f) Evaluate $\begin{vmatrix} 1 & 4 & 16 \\ 1^2 & 2^2 & 4^2 \\ 0 & 1 & 6 \end{vmatrix}$. 2

2. a) Find the extrene points of the function

$$f(x, y) = x^3 + y^3 + 2x^2 + 4y^2 + 6.$$ 7

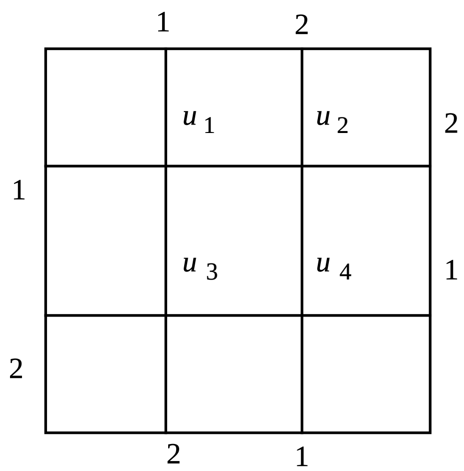
b) Using Picard's method find a solution of $\frac{dy}{dx} = 1 + xy$ up to the third approximation when $y(0) = 0$. 7

3. a) Use Taylor series method to solve the equation

$$\frac{dy}{dx} = -xy, \quad y(0) = 1.$$ 7

b) Apply Runge-Kutta method (fourth order) to find an approximate value of y when $x = 0.2$, given that $\frac{dy}{dx} = x + y$, $y = 1$ when $x = 0$. 7

4. a) Solve $\frac{\partial^2 u}{dx^2} + \frac{\partial^2 u}{dy^2} = 0$ in the domain of the figure given by Gauss-Siedel method : 7



b) Find the extreme points of the function

$f(x, y, z) = x + 2z + yz - x^2 - y^2 - z^2$. 7

5. a) Define analystic function in a region R of the complex plane. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is analystic function of $z = x + iy$, find $f(z)$ in terms of z. 1 + 6

b) State a set of necessary conditions for a function $f(z) = u(x, y) + iv(x, y)$ to be analytic in R.

For the function defined by $f(z) = \sqrt{|xy|}$, show that the Cauchy-Riemann equations are staisfied at (0, 0) but the function is not differentiable at that point. 2 + 5

6. a) Investigate for what values of λ and μ the following equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

i) has no solution

ii) have unique solution

iii) have an inifinite number of solutions ? 7

b) If λ is an eigenvalue of a non-singular matrix A, then prove that λ^{-1} is the eigenvalue of A^{-1}. Hence find the eigenvalues of A^{-1}, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$ 7

7. a) State Cauchy's residue theorem. Use this to evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2}\,dz$, where C is the circle : $|z| = 4$.

2 + 5

b) i) Expand $f(z) = 1/(z+1)(z+3)$ in Laurent's series valid for $0 < |z+1| < 2$. 4

ii) Find the residues of the following function at each pole : $\frac{ze^z}{(z-a)^3}$. 3