Name : ....

Roll No. : ....

Invigilator's Signature : ....

2013

ERROR CONTROL CODING

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *three* from each of Group-B and Group-C.

GROUP – A

Answer *all* questions.

1. a) Define $d_{\min}$ for a convolutional encoder. 2

b) How many different error patterns can *a* (*n*, *k*) linear block code detect ? 1

c) Define cyclic code. 2

d) Define dual spaces. 2

e) Define 'minimal polynomial' of any element of any extension Galois field. 2

f) What are primitive BCH codes ? 1

GROUP – B

Answer any *three* of the following.

2. Determine the generator polynomial of a tripple error correcting (15, 5) BCH code based on GF $\left(2^4\right)$ given in annexure-II. 5

3. Derive a mathematical expression for the probability of undetected word error due a (n, k) block code. Also derive the expression if no channel coding were implemented. 4 + 1

4. What is 'fractional rate loss' in a convolutional code ? Determine the fractional rate loss in a (3, 2, 2) convolutional encoder for a 100 bit long input message. 2 + 3

5. Explain the general structure of Reed-Solomon codes. Also explain that RS code has fairly good burst error correcting capability. 3 + 2

GROUP – C

Answer any *three* of the following.

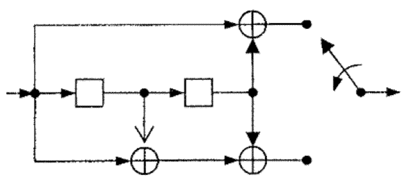
6. a) Show a neat labelled block diagram of Chen's 'search & correct' circuit for a (15, 11), single error correcting BCH code based on the table of GF $\left(2^4\right)$ given in annexure-II. Explain its operation. 5 + 5

   b) Derive a mathematical expression for the probability of undetected word error due a (n, k) block code, terms of the weight enumerator of its dual code. 5

7. a) Determine the generator polynomial of a (15, 11) double error correcting RS code. Also find the code word for the message word u = (11011110). Use GF $\left(2^4\right)$ in annexure-I as reference. 5 + 5

   b) Prepare the state transition table and construct the state diagram for the convolutional coder shown below.

   3 + 2



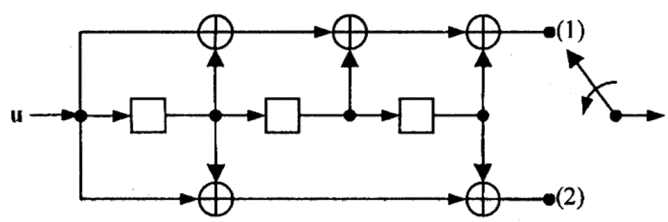
8. a) Establish that all powers of α upto α^{2t} are roots of any code polynomial of a BCH code, where α is the root of the primitive polynomial, on which the code is based. 5

b) Define 'residual bit error rate' and derive a mathematical expression for the residual bit error due a (n, k) block code. 2 + 3

c) When is a block code called 'maximum distance code' ? 2

d) How 'Mc William's identity' relates the weight enumerators of a block code and its dual ? 3

9.



a) Determine the impulse response polynomial $g^{(1)}$ (D) and $g^{(2)}$ (D) for the above convolutional coder. 4

b) For an information sequence u = (10101), determine the two encoded sequence polynomial $v^{(1)}$ (D) and $v^{(2)}$ (D). 4

c) Combine the above two encoded sequence polynomials to determine the code word polynomial v (D) and hence the code word v. 4

d) Establish that all single error correcting BCH codes are Hamming codes. 3

GF $\left(2^4\right)$ Annexure-I $P(X) = 1 + X + X^4$		GF $\left(2^4\right)$ Annexure-II $P(X) = 1 + X^3 + X^4$	
0	0	0	0
1	1	1	1
α	α	α	α
α^2	α^2	α^2	α^2
α^3	α^3	α^3	α^3
α^4	$1 + \alpha$	α^4	$1 + \alpha^3$
α^5	$\alpha + \alpha^2$	α^5	$1 + \alpha + \alpha^3$
α^6	$\alpha^2 + \alpha^3$	α^6	$1 + \alpha + \alpha^2 + \alpha^3$
α^7	$1 + \alpha + \alpha^3$	α^7	$1 + \alpha + \alpha^2$
α^8	$1 + \alpha^2$	α^8	$\alpha + \alpha^2 + \alpha^3$
α^9	$\alpha + \alpha^3$	α^9	$1 + \alpha^2$
α^{10}	$1 + \alpha + \alpha^2$	α^{10}	$\alpha + \alpha^3$
α^{11}	$\alpha + \alpha^2 + \alpha^3$	α^{11}	$1 + \alpha^2 + \alpha^3$
α^{12}	$1 + \alpha + \alpha^2 + \alpha^3$	α^{12}	$1 + \alpha$
α^{13}	$1 + \alpha^2 + \alpha^3$	α^{13}	$\alpha + \alpha^2$
α^{14}	$1 + \alpha^3$	α^{14}	$\alpha^2 + \alpha^3$
β	$\Phi(\beta)$	β	$\Phi(\beta)$
0	X	0	X
1	$1 + X$	1	$1 + X$
$\alpha, \alpha^2, \alpha^4, \alpha^8$	$1 + X + X^4$	$\alpha, \alpha^2, \alpha^4, \alpha^8$	$1 + X^3 + X^4$
$\alpha^3, \alpha^6, \alpha^9, \alpha^{12}$	$1 + X + X^2 + X^3 + X^4$	$\alpha^3, \alpha^6, \alpha^9, \alpha^{12}$	$1 + X + X^2 + X^3 + X^4$
α^5, α^{10}	$1 + X + X^2$	α^5, α^{10}	$1 + X + X^2$
$\alpha^7, \alpha^{11}, \alpha^{13}, \alpha^{14}$	$1 + X^3 + X^4$	$\alpha^7, \alpha^{11}, \alpha^{13}, \alpha^{14}$	$1 + X + X^4$