Name : ....

Roll No. : ....

Invigilator's Signature : ....

2012

ERROR CONTROL CODING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) For (7, 4) Hamming code, the parity-check bits for information words $\left(i_1, i_2, i_3, i_4 \right)$ is

a) $i_1 + i_2 + i_3$ b) $i_1 + i_3 + i_4$

c) $i_1 + i_2 + i_4$ d) both (a) and (b).

ii) For even parity (5, 4) block code, p_c for $p = 0{\cdot}1$ is

a) 0·5905 b) 0·5815

c) 0·5616 d) 0·5915.

iii) In (7, 4) Hamming code, the error syndrome for error pattern '0000010' is

a) (010) b) (100)

c) (001) d) (011).

iv) A block code with minimum distance d_{min} can defect all error patters with error detecting limit, l equal to

a) $d_{min} + 1$ b) $2d_{min} - 1$

c) $d_{min} - 1$ d) $2d_{min} + 1$.

v) Systematic codeword for (5, 3) linear code for information word (101) is

a) (110110) b) (10100)

c) (10101) d) (10011).

vi) The error syndromes of the received word (1101101) in (7, 4) code, is

a) (001) b) (010)

c) (011) d) (100).

vii) If the information polynomial for a (n, k) cyclic code is $i(x)$, generator polynomial is $g(x)$, remainder polynomial is $r(x)$, then the systematic code word polynomial $c(x)$ is equal to

a) $c(x) = i(x) + r(x)$

b) $c(x) = i(x)\, x^{n-k} + r(x)$

c) $c(x) = i(x)\, g(x)$

d) $c(x) = i(x) + g(x)$.

viii) In a (7, 4) code, if the parity check polynomial is $x^4 + x^2 + x + 1$, then the generator polynomial of its dual code is

a) $x^3 + x^2 + x + 1$ b) $x^4 + x^2 + x + 1$

c) $x^4 + x^3 + x^2 + x$ d) $x^4 + x^3 + x + 1$.

ix) The dimension of the subspace in V_5 consisting of the vectors (00000), (11100), (01010), (10001), (10110), (101101) and (00111) is

a) 3 b) 2

c) 4 d) 5.

x) The generator polynomial of GE $\left(2^4 \right)$ is $x^4 + x + 1$. If α^7 is a root of this polynomial belonging to GF $\left(2^4 \right)$, then the minimal polynomial will be

a) $x^4 + x + 1$ b) $x^4 + x^2 + 1$

c) $x^4 + x^3 + 1$ d) $x^3 + x + 1$.

xi) Which of the following codes belongs to non-binary BCH code ?

a) Cyclic code b) Block code

c) Hamming code d) Reed-Solomon code.

xii) Previous inputs are considered in which of the following code ?

a) Convsolation code b) BCH code

c) Reed-Solomon code d) None of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that the following linear codes are not cyclic :

   a) The (6, 3) code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

   b) The (5, 2) code with generator matrix.

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

3. What is Galois field ? Why is it called field ? Give reason and example in support of your answer.

4. What do you mean by vector space ? What are called linear dependent and independent vector spaces ? How can you find the dimension of a vector space and the basis of a vector space ?

5. Find the ninimal polynomials for α, α^2 and α^3 which are the roots of $x^4 + x + 1 = 0$ in GF $\left(2^4 \right)$.

6. Construct a triple error-correcting BCH code with block length $n = 31$ over GF $\left(2^5 \right)$, the generator polynomial in GF $\left(2^5 \right)$ is $g(x) = x^5 + x^2 + 1$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that the row space of the matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

gives the codewords of the (5, 3) linear code.

b) Show that every codeword in the (7, 3) code is orthogonal to every other codeword.

c) What do you mean by dual code ? Give example.

6 + 6 + 3

8. a) Determine whether the polynomials

$p_1(x) = x^4 + x^3 + x + 1$, $p_2(x) = x^2 + x + 1$,

$p_3(x) = x^3 + x^2 + 1$ over GF (2) are (i) irreducible and (ii) primitive.

b) Find the roots of $x^3 + \alpha^8 x^2 + \alpha^{12} x + \alpha = 0$ defined over GF $\left(2^4\right)$.

c) Determine the inverse of the matrix

$$A = \begin{bmatrix} \alpha^{12} & 0 & \alpha \\ 1 & \alpha^8 & \alpha^{14} \\ \alpha^2 & \alpha^{11} & \alpha^5 \end{bmatrix}$$

over GF $\left(2^3\right)$ and GF $\left(2^4\right)$. 6 + 3 + 6

9. a) Determine a linear feedback shift register for dividing by the polymonial $p(x) = x^5 + x^3 + x + 1$. By considering the operation of the register, determine the remainder and quotient of $x^7 + x^2 + 1$ divided by $p(x)$. Check your answer using the 'long hand' method for polynomial division.

b) Draw and explain the operation of an encoder circuit for (7, 4) cyclic code with $g(x) = x^3 + x + 1$. 8 + 7

10. a) Construct a single error-correcting binary BCH code over GF $\left(2^3\right)$.

b) Given the minimal polynomials of α & α^3 in $GF(2^3)$ are $m_1(x) = x^5 + x^2 + 1$ and

$m_3(x) = x^5 + x^4 + x^3 + x^2 + 1$ respectively, construct a binary doubl error-correcting code over $GF(2^5)$.

c) Determine the generator polynomial of the double error-correcting (15, 11) Reed-Solomon codes. 5 + 5 + 5

11. Write short notes on any *three* of the following : 3 × 5

a) Reed-Solomon code

b) Convolutional code

c) Berlekamp algorithm

d) Cyclic code

e) Vector space.