Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/M. Tech (ECE)/SEM-2/MCE-202/2013

2013
ERROR CONTROL CODING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

1. Answer any *ten* of the following : 10 × 1 = 10

   a) A code with minimum distance d(min) = 5. How many errors it can correct ?

   b) Which degree of generator polynomial can generate a (7, 4) Cyclic code ?

   c) What is the factor of a generator polynomial of a Cyclic code ?

   d) What is the memory order of an encoder of (4, 3, 2) convolution code ?

   e) What is the length of a BCH code ?

   f) What are the numbers of shift registers in (2, 1, 2) convolution code ?

g) What is the condition of a dual code in case of Linear Block code ?

h) What are the differences between single error and burst error ?

i) Definition of Block code

j) What is Hamming code ?

k) What is Linear code ?

l) What is Cyclic code ?

GROUP – B

Answer any *three* of the following : $3 \times 5 = 15$

2. A (7, 1) repetition code is used to encode information sent through a channel with a bit-error probability of 0·01. Find the probability that an information bit is erroneous after decoding.

3. Consider the (4, 3) even-parity code with the 8 codewords. (0000), (0011), (0110), (1100), (0101), (1010), (1001), (1111). Show that the code is linear.

4. Consider the (8,4) extended Hamming code. Determine the outcome of a decoder when the codeword c= (00101101) incurs the errors

   a) e_1 = (00100000)

   b) e_2 = (10000100)

   c) e_3 = (00000111)

5. Determine nonsystematic codeword polynomials, for the (7, 4) code, given $i_1 = (1100)$ and $i_2 = (1001)$.

6. Assuming the (7, 4) Cyclic code, determine the systematic codeword polynomials for $i_1 = (1001)$ and $i_2 = (1110)$.

GROUP – C

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Construct a Linear Feedback Shift Register for dividing $x^7 + x^6 + x^2 + x + 1$ by $x^4 + x^2 + 1$.

b) Solve the following equations over the prime field modulo-5

$2x + 3y = 1$

$x + 2y = 2$

c) Given that α is a field element of $GF(2^3)$ evaluate

i) $(\alpha^2 a^{-5} + 1)(\alpha^2 + \alpha)$

ii) $\sqrt{(\alpha^4 \alpha^5 + \sqrt{\alpha})}$.

Repeat when α an element of GF (2^4) is. 6 + 3 + 6

8. a) Determine whether the polynomials

$P1(x) = x4 + x3 + x + 1$

$P2(x) = x2 + x + 1$

$P3(x) = x3 + x2 + 1$

Over GF (2) area (a) irreducible and (b) primitive.

b) Determine systematic and nonsystematic code words for $i = (0\ 1\ 1\ 1)$ given the (7, 4) code with $g(x) = x3 + x + 1$. 8 + 7

9. a) The (15, 11) single-error-correcting binary BCH code has $g(x) = x^4 + x + 1$. Show that α, α^2 and α^4 are roots of $g(x)$ but α^3 is not a root, where α is a primitive field element of GF (2^4). Find the fourth root of $g(x)$.

b) Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & \alpha^4 & \alpha^3 \\ \alpha^2 & 0 & \alpha \\ \alpha^4 & \alpha & \alpha^5 \end{bmatrix}$$

Over GF (2^3) and GF (2^4) 8 + 7

10. a) Construct the (15, 13) single-error-correcting reed-solomon code and determine the systematic codeword corresponding to $i = (0\ 0\ \alpha\ 0\ 0\ 1\ \alpha^7\ \alpha^2\ 0\ 0\ 1\ \alpha\ \alpha^2)$ where α is a primitive element of GF (2^4).

b) Show that α, α^2, α^3 and α^4 are roots of $g(x) = x^8 + x^7 + x^6 + x^4 + 1$, the generator polynomial of the (15, 7) double-error-correcting binary BCH code, where α is a primitive element field of GF (2^4). 8 + 7

11. a) Design a shift register with stages and $g_0 = 1$, $g_1 = 0$, $g_2 = 1$, $g_3 = 1$. Determine the output sequence for the input sequence $u = (1001)$.

b) Determine the output sequence from the (4, 3, 2) convolution encoder, given the input sequences $u^1 = (101)$, $u^2 = (110)$ and $u^3 = (011)$. 8 + 7