



vii) The condition for independence of two events A and B is

- a) $P(A \cap B) = P(A)P(B)$
- b) $P(A + B) = P(A) + P(B)$
- c) $P(A - B) = P(A)P(B)$
- d) $P(A \cap B) = P(A)P(B/A)$.

viii) The probability of the 4 children in a family having different birthdays is

- a) 0.9836
- b) 0.4735
- c) 0.90
- d) 0.757.

ix) The mean of a uniform distribution with parameters a and b is

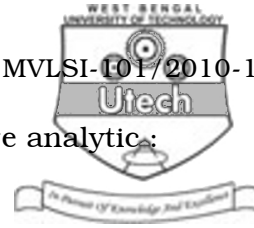
- a) $b - a$
- b) $b + a$
- c) $\frac{a + b}{2}$
- d) $\frac{b - a}{2}$.

x) A random variable X has the following probability density function :

$$f(x) = \begin{cases} 2x & , 0 \leq x \leq 1 \\ 0 & , \text{otherwise} \end{cases}$$

The value of $Var(3 - 5x)$ is

- a) $\frac{1}{18}$
- b) $\frac{25}{18}$
- c) $\frac{12}{18}$
- d) none of these.



5. Prove that the following function is nowhere analytic:

$$f(x + iy) = \sqrt{|xy|} .$$

6. Let a pair of dice be rolled 900 times and X denotes the number of times a total of 9 occurs. Find $P(80 \leq X \leq 120)$, given that $\phi(2.1213) = 0.98$.

7. Use Lagrange's interpolation to find the value of $F(X)$ for $X = 0$, given the following table :

X :	- 1	- 2	2	4
F (X) :	- 1	- 9	11	69

GROUP – C

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Two persons A and B throw alternatively with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his probability of winning.
- b) The probability of a bomb hitting a target is $\frac{2}{5}$. Four direct hits are necessary to destroy a bridge completely. If 6 bombs are aimed at the bridge, what is the probability that the bridge will be destroyed ?
- c) State and prove Baye's theorem. $5 + 5 + 5$



9. a) In a certain factory blades are manufactured in packets of 10. There is a 0.2% probability for any blade to be defective. Using Poisson distribution calculate approximately the number of packets containing two defective blades in a consignment of 20,000 packets.
 ($e^{-0.02} = 0.9802$)
- b) If the equations of two Regression lines obtained in a correlation analysis are $3x + 12y - 19 = 0$ and $9x + 3y = 46$, determine which one is regression equation of y on x and which one is the regression equation of x on y . Find the means of x on y and correlation coefficient between x and y .
- c) In a normal distribution, 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation. Given that $P(0 < Z < 1.405) = 0.42$ and $P(-0.496 < Z < 0) = 0.19$. 5 + 5 + 5
10. a) Determine the largest eigenvalue and the corresponding eigenvector of the matrix :

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} .$$

- b) Compute $\sqrt[3]{2}$ using Newton-Raphson method correct up to 4 significant figures.
- c) Find by suitable interpolation formula the value of y for $x = 0.05$ from the following data :

X :	0.00	0.10	0.20	0.30	0.40
Y :	1.000	1.2214	1.4918	1.8221	2.2255

5 + 5 + 5



11. a) Evaluate $\int_C \frac{4 - 3z}{(z - 1)z(z - 3)} dz$, where C is the circle $|z| = \frac{5}{2}$.

b) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic in C and find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic.

c) Use residue theorem to evaluate

$$\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz, \text{ around the circle } |z| = 2.$$

5 + 5 + 5

12. a) Use dynamic programming to solve

$$\text{Minimize } Z = y_1^2 + y_2^2 + y_3^2$$

$$\text{subject to } y_1 + y_2 + y_3 \geq 15$$

$$\text{and } y_1, y_2, y_3 \geq 0.$$

b) Find the maximum value of $x^3 y^2$ subject to the constraint $x + y = 1$, using the method of Lagrange's multiplier.

7 + 8



13. a) Solve

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0,$$

if $u(0, t) = 0$, $u(x, 0) = e^{-x}$, $x > 0$, $u(x, t)$ is unbounded.

b) If $f(z) = u + iv$ is an analytic function and $z = re^{i\theta}$ where u, v, r, θ are all real. Show that the Cauchy-Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta} \quad \text{and} \quad \frac{\partial u}{\partial \theta} = -r \frac{\partial v}{\partial r}.$$

c) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the point $w = i, 0, -i$.

6 + 5 + 4

