Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH (ECE)/SEM-1/MVLSI-101/2010-11

2010-11

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) If X is normally distributed with zero mean and unit variable, then the expectation of X^2, is

a) 1 b) 0

c) 8 d) 2.

ii) The maximum and minimum values for correlation coefficient are

a) 1, 0 b) 2, 1

c) 0, – 1 d) 1, – 1.

iii) If A and B are two events with $P(A) = 0{\cdot}4$, $P(B) = 0{\cdot}3$ and $P(A \cap B) = 0{\cdot}2$, then $P(A^c \cap B)$ is

a) 0·1 b) 0·2

c) 0·3 d) 0·4.

iv) 50 tickets are serially numbered 1 to 50. One ticket is drawn from these at random. The probability of it being a multiple of 3 or 4 is

a) $\frac{12}{25}$

b) $\frac{6}{25}$

c) $\frac{18}{25}$

d) none of these.

v) The order of the pole $z = 0$ of the function $\frac{\sin z}{z^3}$ is

a) 1 b) 2

c) 3 d) 4.

vi) The value of $\int_C \frac{dz}{z+3}$ (where C is a circle $|z| = 1$) is

a) 0 b) 1

c) 2 d) – 1.

vii) The condition for independence of two events A and B is

a) $P(A \cap B) = P(A)P(B)$

b) $P(A + B) = P(A) + P(B)$

c) $P(A - B) = P(A)P(B)$

d) $P(A \cap B) = P(A)P(B/A)$.

viii) The probability of the 4 children in a family having different birthdays is

a) 0·9836 b) 0·4735

c) 0·90 d) 0·757.

ix) The mean of a uniform distribution with parameters a and b is

a) $b - a$ b) $b + a$

c) $\frac{a+b}{2}$ d) $\frac{b-a}{2}$.

x) A random variable X has the following probability density function :

$$f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

The value of $Var(3 - 5x)$ is

a) $\frac{1}{18}$ b) $\frac{25}{18}$

c) $\frac{12}{18}$ d) none of these.

xi) The number of significant figures in 0·0130583 is

a) seven b) six

c) eight d) five.

xii) The truncation error to compute the integration in Trapezoidal rule is of order

a) h b) h^2

c) h^3 d) h^4 .

GROUP – B

Answer any *three* of the following. $3 \infty 5 = 15$

2. Show that $-1 \le r_{xy} \le 1$, for any bivariate data given by (x, y), where r_{xy} is correlation coefficient of x and y.

3. In answering a question on a multiple choice test, a student either knows the answer or he guesses. Let p be the probability that he knows the answer and $1 - p$ be the probability that he guesses. Assume that a student who guesses the answer will be correct with probability $\frac{1}{n}$. What is the conditional probability that a student knew the answer to a question given that he answered it correctly ?

4. Define non-isolated essential singularity. Show that the function $f(z) = \cot \frac{1}{z}$ has a non-isolated essential singularity at $z = 0$.

5. Prove that the following function is nowhere analytic :

$f(x + iy) = \sqrt{|xy|}$.

6. Let a pair of dice be rolled 900 times and X denotes the number of times a total of 9 occurs. Find $P(80 \le X \le 120)$, given that $\phi(2 \cdot 1213) = 0 \cdot 98$.

7. Use Lagrange's interpolation to find the value of $F(X)$ for $X = 0$, given the following table :

***X* :**	– 1	– 2	2	4
***F* (*X*) :**	– 1	– 9	11	69

GROUP – C

Answer any *three* of the following. 3 ∞ 15 = 45

8. a) Two persons A and B throw alternatively with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his probability of winning.

b) The probability of a bomb hitting a target is $\frac{2}{5}$. Four direct hits are necessary to destroy a bridge completely. If 6 bombs are aimed at the bridge, what is the probability that the bridge will be desroyed ?

c) State and prove Baye's theorem. 5 + 5 + 5

9. a) In a certain factory blades are manufactured in packets of 10. There is a 0·2% probabiliity for any blade to be defective. Using Poisson distribution calculate approximately the number of packets containing two defective blades in a consignment of 20,000 packets. ($e^{-0\cdot02} = 0\cdot9802$)

b) If the equations of two Regression lines obtained in a correlation analysis are $3x + 12y - 19 = 0$ and

$9x + 3y = 46$, determine which one is regression equation of y on x and which one is the regression equation of x on y. Find the means of x on y and correlation coefficient between x and y.

c) In a normal distribution, 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation. Given that $P(0 < Z < 1\cdot405) = 0\cdot42$ and $P(-0\cdot496 < Z < 0) = 0\cdot19$. 5 + 5 + 5

10. a) Determine the largest eigenvalue and the corresponding eigenvector of the matrix :

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

b) Compute $\sqrt[3]{2}$ using Newton-Raphson method correct up to 4 significant figures.

c) Find by suitable interpolation formula the value of y for $x = 0\cdot05$ from the following data :

X :	0·00	0·10	0·20	0·30	0·40
Y :	1·000	1·2214	1·4918	1·8221	2·2255

5 + 5 + 5

11. a) Evaluate $\int_C \frac{4-3z}{(z-1)z(z-3)}\,dz$, where C is the circle $|z| = \frac{5}{2}$.

b) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic in C and find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic.

c) Use residue theorem to evaluate

$$\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)}\,dz,$$ around the circle $|z| = 2$.

5 + 5 + 5

12. a) Use dynamic programming to solve

Minimize $Z = y_1^2 + y_2^2 + y_3^2$

subject to $y_1 + y_2 + y_3 \geq 15$

and $y_1, y_2, y_3 \geq 0$.

b) Find the maximum value of $x^3 y^2$ subject to the constraint $x + y = 1$, using the method of Lagrange's multiplier. 7 + 8

13. a) Solve

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad x > 0, \ t > 0,$$

if $u(0, t) = 0$, $u(x, 0) = e^{-x}$, $x > 0$, $u(x, t)$ is unbounded.

b) If $f(z) = u + iv$ is an analytic function and $z = re^{i\theta}$ where u, v, r, θ are all real. Show that the Cauchy-Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \quad \text{and} \quad \frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}.$$

c) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the point $w = i, 0, -i$.

6 + 5 + 4