Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(ECE-VLSI)/SEM-1/MVLSI-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

*Time Allotted* : 3 Hours   *Full Marks* : 70

Candidates are required to give their answers in their own words
as far as practicable.

Graph sheet is provided by the Institution.

Answer any *five* questions.   5 × 14 = 70

1. a) The chance that a doctor *A* will diagnose a disease *x*
        correctly is 60%. The chance that a patient will die by
        his treatment after correct diagnosis is 40% and the
        chance of death by wrong diagnosis is 70%. A patient of
        doctor *A*, who had disease *x*, died. What is the chance
        that his disease was diagnosed correctly ?

    b) Derive moment generating function of N (0, 1), the
        symbol has its usual meaning and thus derive mean
        and variance. Show the limit of the standardised
        binomial distribution, as *n* tends to ∞, is the standard
        normal distribution.

40352   [Turn over
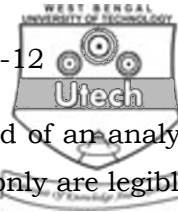
2. a) In a partially destroyed laboratory record of an analysis of correlation data the following results only are legible.

Variance of $x = 9$

Regression equations : $8x - 10x + 66 = 0$

$$40x - 18y = 214$$

What are (i) the mean value of x and y ? (ii) the coefficient of correlation between x and y ? (iii) the standard deviation of y ?

b) i) State Cauchy's Integral theorem.

Evaluate $\oint_C \dfrac{e^z}{(z+1)(z+2)} dz$ where $C : |z-1| = 4$.

ii) Evaluate $\oint_C \dfrac{z \, dz}{(z-1)(z-2)^2}$ where $C : |z-2| = \dfrac{1}{2}$.

3. a) Prove that $u = y^3 - 3x^2 y$ is a harmonic function and find its harmonics conjugate and the corresponding analytic function $f(z)$ in terms of z and thus $f(z)$.

b) i) Find $\displaystyle\int_0^{1+i} \left(x - y + ix^2\right) dz$

ii) Evaluate $\displaystyle\int_0^{1+i} z^2 \, dz$.

c) What kind of singularity have the following functions ?

i) $\cos z - \sin z$ at $z = \infty$

ii) $\dfrac{z^2 + 4}{e^z}$ at $z = \infty$.

4. a) Using Jacobi's method find all the eigenvalues and the eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$$

   b) Using *R-K* method of order four, solve $y'' = y + xy'$, $y(0) = 1$, $y'(0) = 0$ to find $y(0 \cdot 2)$ and $y'(0 \cdot 2)$.

5. a) Classify the equations :

   i) $u_{xx} + 2u_{xy} + u_{yy} = 0$

   ii) $(1 + x^2) u_{xx} + (5 + 2x^2) u_{xt} + (4 + x^2) u_{tt} = 0$

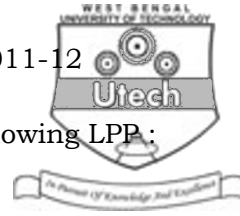   b) Solve by Crank-Nicolson's method

   $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad \text{for } 0 < x < 1, \ t > 0$$

   given that $u(0, t) = 0, u(1, t) = 0$ and $u(x, 0) = 100(x - x^2)$.

   Compute u for one time step with $h = \dfrac{1}{4}$.

6. a) Define the following terms 'stage', 'state', 'principle of optimality'.

   b) Find the maximum of the sum of the squares of the three positive integers whose product does not exceed 4, using dynamic programming.

7. Using dynamic programming, solve the following LPP :

Maximize $Z = 3x_1 + 5x_2$

subject to $x_1 \leq 4$

$x_2 \leq 6$

$3x_1 + 2x_2 \leq 18$

$x_1, x_2 \geq 0$

=============