



Name :

Roll No. :

Invigilator's Signature :

CS/M.TECH (ECE)/SEM-1/MCE-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

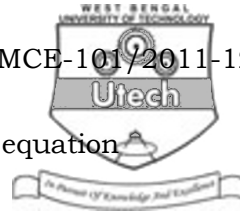
Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. a) What is difference between a closed set of complex numbers and a closed set of real numbers ? Illustrate with examples. 3
- b) Prove that the zeros of $f(z) = \sin(z)$ are all real. 3
- c) Prove that the derivative of the function $\sin(z)$ is $\cos(z)$ 2
- d) Find the eigenvalues of the matrix $\begin{vmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{vmatrix}$ 3
- e) Verify the consistency of the following system of equations :
 $2x + 3y + 4z = 11, x + 5y + 7z = 15, 3x + 11y + 13z = 25$ 3



2. a) Find the residues of the function $\frac{\cot(\pi z)}{(z-a)^2}$ 7 + 7
 b) Evaluate $\int_0^{2\pi} \frac{dx}{1+a^2-2a\cos x}$ where $0 < a < 1$
3. a) For the function $f(z) = \sec(1/z)$, locate and name the singularities in the finite z -plane and determine whether they are isolated singularities or not.
 b) Locate and name the singularities of $f(z) = \frac{z^8 + z^4 + 2}{(z-1)^3(3z+2)^2}$
 c) Prove that $\lim_{z \rightarrow 0} \left(\frac{\bar{z}}{z}\right)$ does not exist. 6 + 4 + 4
4. a) Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $z = x + y$
 b) Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225, z = 0$ 9 + 5
5. a) Determine the maximum and minimum values of the function $f(x) = 12x^5 - 45x^4 + 40x^3 + 5$
 b) Suppose P and Q are two points in the plane. Imagine there is a thin, flexible wire connecting the two points. Suppose P is above Q and we let a frictionless bead travel down the wire impelled by gravity alone. By changing the shape of the wire we might alter the amount of time it takes for the bead to travel from P to Q. What shape should the wire take in order to make the travel time of the bead smallest? 4 + 10



6. a) Use Taylor series method to solve the equation

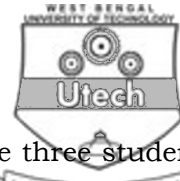
$$\frac{dy}{dx} = -xy, \quad y(0) = 1$$
- b) Apply Runge-Kutta method (fourth order) to find an approximate value of y when $x = 0.2$, given that

$$\frac{dy}{dx} = x + y^2 \text{ and } y = 1 \text{ when } x = 0$$
7 + 7
7. a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the domain of the figure given by Gauss-Siedal method

	1	2
2	u_1	u_2
	u_3	u_4
	2	1

- b) Solve the system of non-linear equations

$$x^2 + y = 11, \quad y^2 + x = 7,$$
 taking an initial approximation as $x_0 = 3.5$ and $y_0 = -1.8$.
 7 + 7
8. a) Three identical boxes contain red and white balls. The first box contains 3 red and 2 white, the second box 4 red and 5 white and the third box 2 red and 4 white balls. A box is chosen at random and a ball is drawn from it. If the ball drawn is red, what is the probability that the second box is chosen ?



- b) The probabilities of solving a problem by three students A , B and C are $\frac{2}{7}$, $\frac{3}{8}$ and $\frac{1}{2}$ respectively. If all of them try independently, find the probability that the problem is solved.
- c) Given $P(A) = 3/8$, $P(B) = 5/8$ and $P(A \cup B) = 3/4$; find $P(A/B)$ and $P(B/A)$. Are the two events A and B independent ?
9. a) The probability of a bomb hitting a target is 0.2. Two direct hits are necessary to destroy a bridge. If six bombs are aimed at the bridge, find the probability that the bridge is destroyed.
- b) Fit a linear regression of marks in University Examination to the same in college test.

6 + 4 + 4

7 + 7

Serial No.	1	2	3	4	5	6
Marks in College test	35	42	20	50	72	64
Marks in University examination	40	48	24	60	84	68