

Name :

Roll No. :

Invigilator's Signature :

CS/M.TECH (ECE)/SEM-1/MVLSI-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Very Short Type Questions)

1. Answer any *five* of the following questions : 5 × 2 = 10

- a) Evaluate $1 - e^{-hD} \int \square$.
- b) If $\frac{4}{3}$ is represented by the approximate number 1.3333, compute absolute, relative and percentage errors.
- c) The p.d.f. of a random variable X is
$$f(x) = cx^2, 0 \leq x \leq 1.$$
Find (i) c (ii) $P\left(0 \leq x \leq \frac{1}{2}\right)$.
- d) State Beltrami's Identity.
- e) State Bayes' Theorem.
- f) Distinguish between mutually exclusive events and independent events with an example.



- g) If $f(x, y, z) = \sqrt{\frac{1+y^2}{z^2}}$, compute f_x, f_z .
- h) State two different situations where classical definition of probability fails.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Two urns contain respectively 2 red, 5 black, 7 green and 1 red 4 black, 9 green balls. One ball is drawn from each urn. Find the probability that both the balls are of the same colour.
3. Find the singularities of the function $f(z) = \sec \frac{1}{z}$ in the finite z -plane and give the nature of singularities.
4. Find the mean and variance of Binomial distribution.
5. Assuming that the height distribution of a group of men is normally, find the mean and standard deviation, if 84% of the men have heights less than 65.2 inches and 68% have height lying between 65.2 and 62.8 inches.
6. Find for which values of x the following function is maximum and minimum ?

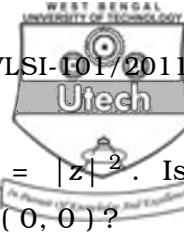
$$f(x) = \frac{x^2 + x + 1}{x^2 - x + 1}.$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. State and prove 'Euler-Lagrange' equation.



8. a) Define $f : \mathbb{C} \rightarrow \mathbb{C}$ by $f(z) = |z|^2$. Is it differentiable at $(0, 0)$? Is it analytic at $(0, 0)$? 5

- b) Consider the function f defined as

$$f(z) = \begin{cases} xy(y - ix), & \text{for } z \neq 0 \\ 0, & \text{for } z = 0 \end{cases}$$

Show that f satisfies C-R Equation at the origin, but it is not analytic there. 5

- c) If f is analytic on a domain $S \subseteq \mathbb{C}$. Prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2. \quad 5$$

9. a) Using Newton's divided difference formula evaluate $f(8)$, given that

$x:$	4	5	7	10	11	13
$f(x):$	48	100	294	900	1210	2028

5

- b) Find the smallest positive root of the equation

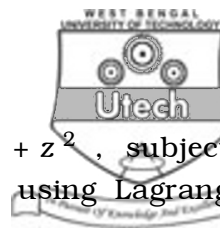
$3x^3 - 9x^2 + 8 = 0$, correct up to four decimal places, using Newton-Raphson method. 5

- c) Use Runge-Kutta method of the fourth order to find $y(0, 1)$, given that

$$\frac{dy}{dx} = \frac{1}{x+y}, \quad y(0) = 1. \quad 5$$

10. a) If $F[y(x)] = \int_a^b \sqrt{1 + [y'(x)]^2} dx$, then using

Euler Lagrange's equation prove that the solution is a straight line. 6



- b) Find the minimum value of $x^2 + y^2 + z^2$, subject to the condition $2x + 3y + 5z = 30$, using Lagrange's method of undetermined multipliers. 9

11. a) Find numerically the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}, \text{ by Power Method and hence find the remaining eigenvalues.} \quad 7$$

- b) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ by Gaussian method. 8

12. a) Prove that

$$\int_L \bar{z} |z| dz = r^3 \pi i$$

where L is the curve consisting of the half-circle $z = re^{ie}$, $0 \leq t \leq \pi$ and the straight line segment $-r \leq \text{Re}(z) \leq r$, $\text{Im}(z) = 0$. 5

- b) Evaluate :

$$\int_C \frac{dz}{z^2 + 1}$$

Where C is the circle (i) $|z + i| = 1$, (ii) $|z - i| = 1$. 5

- c) Find Laurent series corresponding to the function

$$f(z) = \frac{e^z}{z - z^2}$$

That converges for $0 < |z| < R$ and determine its precise region of convergence. 5