Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH (ECE)/SEM-1/MVLSI-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Very Short Type Questions)

1. Answer any *five* of the following questions : $5 \times 2 = 10$

   a) Evaluate $1 - e^{-hD} \int \square$.

   b) If $\frac{4}{3}$ is represented by the approximate number $1 \cdot 3333$, compute absolute, relative and percentage errors.

   c) The p.d.f. of a random variable X is

      $$f(x) = cx^2 \ , 0 \le x \le 1 .$$

      Find (i) c (ii) $P\left(0 \le x \le \frac{1}{2}\right)$.

   d) State Beltrami's Identity.

   e) State Bayes' Theorem.

   f) Distinguish between mutually exclusive events and independent events with an example.

g) If $f(x, y, z) = \sqrt{\dfrac{1 + y^2}{z^2}}$, compute f_x, f_z.

h) State two different situations where classical definition of probability fails.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Two urns contain respectively 2 red, 5 black, 7 green and 1 red 4 black, 9 green balls. One ball is drawn from each urn. Find the probability that both the balls are of the same colour.

3. Find the singularities of the function $f(z) = \sec \dfrac{1}{z}$ in the finite z-plane and give the nature of singularities.

4. Find the mean and variance of Binomial distribution.

5. Assuming that the height distribution of a group of men is normally, find the mean and standard deviation, if 84% of the men have heights less than 65·2 inches and 68% have height lying between 65·2 and 62·8 inches.

6. Find for which values of x the following function is maximum and minimum ?

$$f(x) = \frac{x^2 + x + 1}{x^2 - x + 1} .$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. State and prove 'Euler-Lagrange' equation.

8. a) Define $f : \mathbb{C} \varnothing \mathbb{C}$ by $f(z) = |z|^2$. Is it differentiable at (0, 0) ? Is it analytic at (0, 0) ? 5

b) Consider the function f defined as

$$f(z) = \begin{cases} xy(y - ix), & \text{for } z \neq 0 \\ 0, & \text{for } z = 0 \end{cases}$$

Show that f satisfies C-R Equation at the origin, but it is not analytic there. 5

c) If f is analytic on a domain $S \subseteq C$. Prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4\,|f'(z)|^2 .$$ 5

9. a) Using Newton's divided difference formula evaluate $f(8)$, given that

x :	4	5	7	10	11	13
f (x) :	48	100	294	900	1210	2028

5

b) Find the smallest positive root of the equation

$3x^3 - 9x^2 + 8 = 0$, correct up to four decimal places, using Newton-Raphson method. 5

c) Use Runge-Kutta method of the fourth order to find $y(0, 1)$, given that

$$\frac{dy}{dx} = \frac{1}{x+y}, \quad y(0) = 1.$$ 5

10. a) If $F[y(x)] = \int_a^b \sqrt{1 + [y'(x)]^2}\, dx$, then using Euler Lagrange's equation prove that the solution is a straight line. 6

b) Find the minimum value of $x^2 + y^2 + z^2$, subject to the condition $2x + 3y + 5z = 30$, using Lagrange's method of undetermined multipliers. 9

11. a) Find numerically the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$, by Power Method and hence find the remaining eigenvalues. 7

b) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ by Gaussian method. 8

12. a) Prove that

$$\int_L \bar{z} \; |z| \; dz = r^3 \; \pi i$$

where L is the curve consisting of the half-circle $z = re^{ie}$, $0 \le t \le \pi$ and the straight line segment $-r \le Re(z) \le r$, $lm(z) = 0$. 5

b) Evaluate :

$$\int_C \frac{dz}{z^2 + 1} \; .$$

Where C is the circle (i) $|z + i| = 1$, (ii) $|z - i| = 1$. 5

c) Find Laurent series corresponding to the function

$$f(z) = \frac{e^z}{z - z^2}$$

That converges for $0 < |z| < R$ and determine its precise region of convergence. 5