

## CS/M.TECH (ECE) VLSI/SEM-1/MVLSI-101/2010-11 2010-11 <br> ADVANCED ENGINEERING MATHEMATICS

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

## ( Very Short Answer Type Questions )

1. Answer all of the following questions :

$$
7 \times 2=14
$$

i) The Newton-Raphson method is used to find the root of the equation $x^{2}-2=0$. If the iteration started from -1 , then where the iteration will converge ?
ii) In method of Bisection for the equation $f(\mathrm{x})=0, \mathrm{a} \leq \mathrm{x} \leq b$ find the length of the interval after $n$ iteration.
iii) Four fair coins are tossed simultaneously. Find the probability that at least one head and one tail turn up.
iv) Find the residue of the function $f(z)=\frac{5 z}{(z-1)(z-2)}$ at $z=1$.

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v) If $3 \frac{\mathrm{~d} y}{\mathrm{~d} x}+5 y^{2}=\sin \mathrm{x}, \mathrm{y}(0.3)=5$, using a step size of

$h=0.3$, find the value of $y(0.9)$ using Euler's method.
vi) Find the minimum value of the function $f(x)=e^{x}+e^{-x}$, where $x$ is real number.
vii) Evaluate $\oint_{|z|=1} \frac{\sin z}{z} \mathrm{~d} z$

## GROUP - B <br> ( Long Answer Type Questions )

Answer any four of the following questions :

$$
4 \times 14=56
$$

2. a) The velocity ( $\mathrm{m} / \mathrm{s}$ ) of a body is given as a function of time (seconds) by

$$
v(t)=200 \ln (1+t)-t, t \geq 0
$$

Using Euler's method with a step size of 5 seconds, find the distance in metres travelled by the body from $t=2$ to $t=12$ seconds.
b) Evaluate $\sqrt{11}$ to three places of decimals by NewtonRaphson method.
c) Prove that Newton-Raphson method has a quadratic convergence.5
3. a) State and prove Bayes theorem. 6

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b) Two identical urns contains respectively white, 7 black balls and 4 white, 2 black balls. An urn is selected at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn?
c) The distribution function $F_{X}(x)=\left\{\begin{array}{l}0,-\infty<x<0 \\ \frac{1}{5}, 0 \leq x<1 \\ \frac{3}{5}, 1 \leq x<3 \\ 1, \quad x \geq 3\end{array}\right.$

Find the values of $P(X=1)$ and $P(X=-1)$.
4. a) Show that $f(z)=|z|^{2}$ is continuous everywhere but it is nowhere differentiable except origin.
b) Evaluate $\oint_{|z|=4} \frac{z+1}{\left(z^{2}-2 z\right)} \mathrm{d} z$
c) Evaluate $\oint_{|z|=4} \frac{z}{(z-1)(z-2)^{2}} \mathrm{~d} z$ by residue theorem. 5
5. a) If $x y z=a b c$, show that the maximum value of $b c x+c a y+a b z$ is $3 a b c$ by Lagrange's method of Multipliers.
b) Show that $f(x, y, z)=(x+y+z)^{3}-3(x+y+z)-24 x y z$ has a minimum at (1, 1, 1,) and a maximum at $(-1,-1,-1)$.
6. a) Solve the recurrence relation $f_{n}=f_{n-1}+f_{n-2}$ for $n \geq 2$, $f_{0}=0$ and $f_{1}=1$.

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b) Find the bilinear transformation, which mapsthe points $z=1,0,-1$ onto the points $w=i, 0,-i$. Also find the fixed points of the transformation.
c) The probability density function of a random variable $X$ is $f(x)=k(x-1)(2-x)$ for $1 \leq x \leq 2$. Determine -
i) the value of the constant $k$
ii) $\quad P\left(\frac{5}{4} \leq X \leq \frac{3}{2}\right)$.
7. a) Write the Kuhn-Tucker conditions for the following minimization problem : Min. $f(x)=x_{1}^{2}+x_{2}^{2}+x_{3}^{2}$, subject to $2 x_{1}+x_{2} \leq 5, x_{1}+x_{3} \leq 2, x_{1} \geq 1, x_{2} \geq 2$ and $x_{3} \geq 0$.
b) Prove the condition of convergence for Newton-Raphson method to solve an transcendental equation $\left|f(x) \cdot f^{\prime \prime}(x)\right| \leq\left(f^{\prime}(x)\right)^{2}$. 4
c) From the following table, find the value of $f(1.5)$ by Newton's forward interpolation formula or Lagrange's interpolation :

| $x$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 5 | 10 | 15 | 20 | 25 |

