



Name :
Roll No. :
Invigilator's Signature :

CS/M.Tech (ECE-VLSI)/SEM-1/MVLSI-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

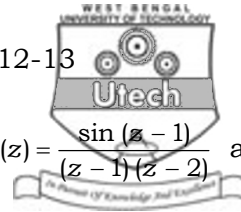
The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Short Answer Type Questions)

1. Answer the following questions : 7 × 2 = 14
- a) How many iterations do you need in bisection method to get the root if you start with $a = 1$ and $b = 2$ and the tolerance is 10^{-4} ?
 - b) If $f(z) = \frac{\sin^3 z}{z^{2012}}$ then find the poles and the order of the poles.
 - c) The Newton-Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If the iteration started from -1 , then where the iteration will converge ?
 - d) A die is rolled three times. Find the probability that exactly one odd number turns up among the three outcomes.



- e) Find the residue of the function $f(z) = \frac{\sin(z-1)}{(z-1)(z-2)}$ at $z = 1$.
- f) Find the points of local maxima and local minima of the function $f(x) = \sin 2x$ in $[-\pi, \pi]$.
- g) Evaluate $\oint_{|z|=1} \frac{\sin z}{z(z-1)} dz$.

GROUP - B

(Long Answer Type Questions)

Answer any *four* of the following. $4 \times 14 = 56$

2. a) Show that $f(z) = |z|^2$ is continuous everywhere but it is nowhere differentiable except origin. 5
- b) Evaluate $\int_{|z+i|=3} \frac{\sin z + \cos z}{z^2 - 1} dz$. 5
- c) $\int_{|z|=4} \frac{z}{(z-1)(z-2)^2} dz$ by Cauchy's integral formula. 4
3. a) Find a positive root of $x^3 - 3x + 1$ using method of bisection method. 5
- b) Evaluate $\sqrt[3]{65}$ to three places of decimals by Newton-Raphson method. 5
- c) Define order or convergence of an iterative method. Prove that bisection method is a linearly convergent. 4



4. a) State and prove Bayes' theorem. 6
- b) An unbiased coin is tossed repeatedly until the outcome of two successive tosses is the same. Assuming that the trials are independent, find the expected number of tosses. 5

c) The distribution function $F_x(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{6}, & 0 \leq x < 1 \\ \frac{3}{6}, & 1 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$

Find the values of $P(X = 1)$ and $P(x = -1)$. 3

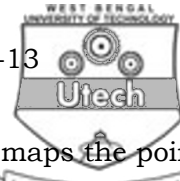
5. a) State and prove the Cauchy-Goursat theorem for complex valued function. 6
- b) What is the probability that in a randomly chosen group of r people, no people have the same birthday? (Consider one year = 365 days). 4

c) Evaluate $\int_i^{2-i} (xy + ix) dz$ along the straight line joining $z = i$ and $z = 2 - i$. 4

6. a) What is cubic spline? Find the interpolating polynomial using cubic spline for the following data: 9

x	-1	0	1
$Y = f(x)$	1	2	-1

- b) In a population of N families, 50% of families have three children, 30% of the families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children? 5



7. a) Find the bilinear transformation, which maps the points $z = 1, 0, -1$ onto the points $w = i, 0, -i$. Also find the fixed points of the transformation. 5
- b) Two identical urns contain respectively 7 white, 7 black balls and 4 white, 4 black balls. An urn is selected at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn? 5
- c) If $f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ is the density function of a continuous random variable X . Find the value of k and the distribution function $F_x(x)$. 4
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