Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.TECH(ECE)VLSI/SEM-1/MVLSI-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. 5 × 14 = 70

1. Consider the continuous time function

$x(t) = \cos(20 * pi * t) + \cos(100 * pi * t)$.

a) What are the corresponding frequencies at continuous and discrete domain ?

b) What should be the minimum sampling frequency ?

c) Write the expression for the equivalent discrete signal $x(n)$.

d) Find the *Z*-transform of (i) $a \wedge n\, u(n)$, (ii) $\cos(w_0 * n)u(n)$. 2 + 2 + 2 + 4 + 4

2. Find $p(x \le a/3)$, $p(-a/2 \le x \le a/4)$, $p(x > 0)$, $p(x \le a/3$ and $x > a/2)$ for the following probability distribution :

   $F(x) = h(s + x^2)^3$ for $-a < x \le a$; where h, s are constants.

   Find $p(x \le a/4 | x \le a/2)$. Demonstrate Bayes theorem for the above distribution with any suitable range of x and corresponding splitting of the domain of x into disjoint sets.

   7 + 7

3. You have to find the increase of the probability of signals of sparks under the following :

   Signal of a spark is generated if I fixed queue points are joined to all of them in the set in any definite order without joining to any of the other-than-I queue points. Knowing that there are d queue points other than the I and queue points joined together in all orders can form, fit the condition that when k points one after the other belong to the set of any d of the I fixed points, the next point connects in a way so as to necessitate the next steps to be in the I-d set. If the steps are equi-probable, calculate the probability to show that any s fixed queue points chosen at random, with n non-I points satisfy a well-known discrete probability distribution.

4. Dock 1 to dock 4 can be the string for sending and redirecting several wedding gifts to different places. Join the dock-vertices to different sending-addresses to generate a graph that webs the following relations : eight gifts have six directed ones but two others need to be sent to any of the five

addresses from the six before or a seventh one. In the dock-list limits are given on that dock1 can send 3 gifts, dock2 sends 1, dock3 gets 2 and dock4 gets the remaining 2. Choose any two different docks to have the ones to be redirected as follows : send the first one to the address/es where that dock has sent its gifts and then keep sending it to all the rest in sequences of all even followed by all odd dock numbers until it reaches the seventh address. Then send the second to any of the even docks' addresses and then to the seventh address. Provide the digraph generated including the relations that would be relevant to produce antisymmetry property and transitivity.

5. a) A pair of dice is tossed thrice. Find the probability of not getting a sum of 6 or 10 in any of the three tosses.

   b) Write the regression lines of Y on X and X on Y. Show that the correlation coefficient of X and Y is the geometric mean of the two regression coefficients. 7 + 7

6. a) Show that the number of vertices of odd degree in any graph is always even.

   b) In a regular graph G with 40 edges, the degree of each vertex is 5. Find the number of vertices in G.

   c) If possible draw a simple graph with 4 vertices and 7 edges. If not explain why. 7 + 4 + 3

7. a) Integrate : $\oint \frac{dz}{(z*2+z+1)^2}$

over a contour that contains the singularities.

b) Integrate : $\int_{-\infty}^{+\infty} \frac{dx}{(x^2-3)^4}$ 7 + 7

8. Consider the sequence

$$x(n) = \text{delta}(n) + 2 * \text{delta}\ (n-2) + \text{delta}\ (n-3)$$

a) Find out 4-point DFT of $x(n)$.

b) Find out $X(z)$.

c) Find out third and fifth harmonics of a sawtooth wave of duration 50 ms and peak value 5 units.

d) Find out the *dc* component. 3 + 3 + 5 + 3

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