Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.TECH (ECE-OLD)/SEM-1/MCE-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS AND STATISTICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Answer Q. No. 1 and any *four* questions from the rest.

1. Answer the following questions : $7 \times 2 = 14$

   a) Define universal set and a complement of a given set with examples.

   b) How does the difference equation,

   $u_{x+2} - (a + b) u_{x+1} + ab\, u_x = 0$,

   arise from the relation $U_x = Aa^x + Bb^x$, A and B being two arbitrary constants ?

   c) Describe singular point of a complex function and the concept of its poles.

d) If $w = f(z) = u(x, y) + iv(x, y)$ be an analytic function is some region of the z-plane, then show that $\nabla^2 u = \nabla^2 v = 0$.

e) Write down the form in which Newton's formula for forward interpolation is usually written for a function $y = \phi(x)$.

f) Describe the classical definition of probability and discusss its limitation.

g) Explain the measure of central tendency of frequency distribution.

2. a) If a finite set has n elements, then prove that it has 2^n subsets.

b) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

6 + 8

3. a) Determine the analytic function whose real part is

$$x^3 - 3xy^2 + 3x^2 - 3y^2 + 2x + 1.$$

b) Show that $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz = 4(\pi + 1)i$,

where c is the circle, $|z| = 3$. 7 + 7

4. a) Find Newton's formula for forward interpolation in terms of x.

b) From the table given below, calculate $\phi(1, 2)$ correct to two decimal places.

x :	0	1	2	3	4
$\phi(x)$:	1·00	1·50	2·20	3·10	4·60

8 + 6

5. a) Find the extremal of the function $\int_{x_0}^{x_1} \frac{y^2}{x^3} dx$.

b) Discuss Lagrange's multiple method to solve constrained problems of optimization. 6 + 8

6. a) The sum of two non-negative quantities is equal to $2n$. Find the probability that their product is not less than $\frac{3}{4}$ times their greatest product.

b) Find the mean age from the following distribution :

Age in years :	15-19	20-24	25-29	30-34	35-39	40-44
No. of persons :	37	81	43	24	9	6

7 + 7

7. a) Define residue of a function $f(z)$ at its singularity z_0. Assuming Laurent's expansion of $f(z)$ in the neighbourhood of z_0, find its residue at z_0.

b) Determine the poles and residues of the function :

$$F(z) = \frac{1}{z^4 + 2z^2 + 1} .$$ 7 + 7