Name :	
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Invigilator's Signature :	

CS/M.TECH(ECE)/SEM-1/MCE-101/2011-12

2011

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks: 70

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any four from the rest.

- a) If A and B are two independent events then show that 1. A and B^c are also independent. 4
 - b) If $f(x,y) = \sqrt{|xy|}$ show that $f_{x}(x,y) = \begin{cases} \frac{1}{2}\sqrt{\frac{|y|}{|x|}} & \text{if } x > 0\\ -\frac{1}{2}\sqrt{\frac{|y|}{|x|}} & \text{if } x < 0 \end{cases}$ and $f_y(x,y) = \begin{cases} \frac{1}{2}\sqrt{\frac{|x|}{|y|}} & \text{if } y > 0\\ -\frac{1}{2}\sqrt{\frac{|x|}{|y|}} & \text{if } y < 0 \end{cases}$ 5
 - c) Evaluate $\int_{a} \frac{dz}{(z-a)^n}$ for n = 2, 3, 4 ... where c is a closed 5

curve containing the point z = a.

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- 2. a) Examine the maxima and minima of the function $f(x,y) = 2x^2 xy + 2y^2 20x$.
 - b) Let y = F (x,t), where F is a differentiable function of two independent variables x and t which are related to variables u and v by the relations u = x + ct, v = x ct.

Prove that $\frac{\partial^2 y}{\partial^2 x^2} - \frac{1}{c^2} \frac{\partial^2 y}{\partial^2 t^2} = 0$ can be transformed into $\frac{\partial^2 y}{\partial u \partial u} = 0.$ 7

- 3. a) Find the stationary points of $f(x, y, z) = x^2y^2z^2$ subject to the condition $x^2+y^2+z^2=a^2$, where x, y, z are positive. Also find the maximum value of the same function subject to the condition $x^2 + y^2 + z^2 = a^2$ by application of Lagrangian multiplier method. 7
 - b) If f(0) = 0 and $f'(x) \frac{1}{1+x^2}$ then prove without using method of integration that $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$. 7
- 4. a) The value of sinx for different values of x are given below. Form a difference table and from this table find sin32° and sin53° using proper formula.

x°	30	35	40	45	50	55
sin x	0.5000	0.5736	0.6428	0.7071	0.7660	0.8192

b) Find the polynomial f(x) and hence calculate f(5.5) for the given data 7

x	0	2	3	4	7
<i>f</i> (<i>x</i>)	1	47	97	251	477

- 5. a) Find the convergence of the Newton-Raphson method. Using Newton-Raphson method, obtain iteration formula for the reciprocal of a number N and hence find the value of $\frac{1}{22}$, correct to three significant figures. 7
 - b) Using the modified Euler's method find y (1·2) where $\frac{dy}{dx} = \frac{x+y}{2}$, y(1) = 3.595 and h = 0.1 7

6. a) Expand
$$f(z) = \frac{1}{z^2(z-i)}$$
 as a Laurent's series about *i*
and hence find the residue there. 7

b)
$$\int_{0}^{2\pi} \frac{1+\sin\theta}{3+\cos\theta} d\theta \text{ using method of residues.}$$
7

7. a) Evaluate
$$\int_{0}^{2+i} (\overline{z})^{2} dz$$
 along the following paths :

i) the straight line
$$y = \frac{x}{2}$$

- ii) first along the real axis to 2 and then vertically to (2 + i). 7
- b) Determine the analytic function whose real part is $e^{x}(x\cos y y\sin y)$. 7
- 8. a) X and Y stand in a queue at random with 10 other people. What is the probability that there are exactly 3 people between X and Y?7
 - b) There are 3 good and 1 bad coins. The bad one has head on both sides. A coin is chosen randomly and tosses 4 times. If head occurs all the 4 times what is the probability that the bad coin has been chosen for toss ?

3

7

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- 9) a) If the daily wage of 10000 workers in a factory follows normal distribution with mean and standard deviation of Rs. 70 and Rs. 5 respectively, find the expected number of workers whose daily wages are :
 - i) between Rs. 66 and Rs 72
 - ii) more than Rs. 72.

Here it is given that : $\frac{1}{\sqrt{2\pi}} \int_{0}^{0} e^{-t^{2}/2} dt = 0.1554$ and $\frac{1}{\sqrt{2\pi}} \int_{0}^{0.8} e^{-t^{2}/2} dt = 0.2881$ 7

b) Let X denote the number of misprints on a page in a certain book. Assume that the random variable X follows Poisson distribution. If $E(X^2) = 6$ then find out the probability that a randomly chosen page will have at least one misprint. 7

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