

CS/M.TECH(ECE)/SEM-1/MCE-101/2011-12

## 2011

## ADVANCED ENGINEERING MATHEMATICS

Time Allotted: 3 Hours
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any four from the rest.

1. a) If $A$ and $B$ are two independent events then show that $A$ and $B^{c}$ are also independent.
b) If $f(x, y)=\sqrt{|x y|}$ show that
$f_{x}(x, y)= \begin{cases}\frac{1}{2} \sqrt{\frac{|y|}{|x|}} \text { if } x>0 \\ -\frac{1}{2} \sqrt{\frac{|y|}{|x|}} \text { if } x<0\end{cases}$
and $f_{y}(x, y)= \begin{cases}\frac{1}{2} \sqrt{\frac{|x|}{|y|}} \text { if } y>0 \\ -\frac{1}{2} \sqrt{\frac{|x|}{|y|}} \text { if } y<0\end{cases}$
c) Evaluate $\int_{c} \frac{\mathrm{~d} z}{(z-a)^{n}}$ for $n=2,3,4 \ldots$ where c is a closed curve containing the point $z=a$.
2. a) Examine the maxima and minima of the function
$f(x, y)=2 x^{2}-x y+2 y^{2}-20 x$.

b) Let $y=F(x, t)$, where $F$ is a differentiable function of two independent variables $x$ and $t$ which are related to variables $u$ and $v$ by the relations $u=x+c t, v=x-c t$.
Prove that $\frac{\partial^{2} y}{\partial^{2} x^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} y}{\partial^{2} t^{2}}=0$ can be transformed into $\frac{\partial^{2} y}{\partial u \partial v}=0$. 7
3. a) Find the stationary points of $f(x, y, z)=x^{2} y^{2} z^{2}$ subject to the condition $x^{2}+y^{2}+z^{2}=a^{2}$, where $x, y, z$ are positive. Also find the maximum value of the same function subject to the condition $x^{2}+y^{2}+z^{2}=a^{2}$ by application of Lagrangian multiplier method.
b) If $f(0)=0$ and $f^{\prime}(x)-\frac{1}{1+x^{2}}$ then prove without using method of integration that $f(x)+f(y)=f\left(\frac{x+y}{1-x y}\right) .7$
4. a) The value of $\sin x$ for different values of $x$ are given below. Form a difference table and from this table find $\sin 32^{\circ}$ and $\sin 53^{\circ}$ using proper formula. 7

| $x^{\circ}$ | 30 | 35 | 40 | 45 | 50 | 55 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sin x$ | 0.5000 | 0.5736 | 0.6428 | 0.7071 | 0.7660 | 0.8192 |

b) Find the polynomial $f(x)$ and hence calculate $f(5 \cdot 5)$ for the given data

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| $x$ | 0 | 2 | 3 | 4 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 1 | 47 | 97 | 251 | 477 |


5. a) Find the convergence of the Newton-Rapksonamethod. Using Newton-Raphson method, optain iteration formula for the reciprocal of a number $N$ and hence find the value of $\frac{1}{22}$, correct to three significant figures. 7
b) Using the modified Euler's method find $y(1 \cdot 2)$ where $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{x+y}{2}, y(1)=3 \cdot 595$ and $h=0 \cdot 1$ 7
6. a) Expand $f(z)=\frac{1}{z^{2}(z-i)}$ as a Laurent's series about $i$ and hence find the residue there.
b) $\int_{0}^{2 \pi} \frac{1+\sin \theta}{3+\cos \theta} d \theta$ using method of residues.
7. a) Evaluate $\int_{0}^{2+i}(\bar{z})^{2} \mathrm{~d} z$ along the following paths:
i) the straight line $y=\frac{x}{2}$
ii) first along the real axis to 2 and then vertically to $(2+i)$.
b) Determine the analytic function whose real part is $e^{x}(x \cos y-y \sin y)$.
8. a) $X$ and $Y$ stand in a queue at random with 10 other people. What is the probability that there are exactly 3 people between $X$ and $Y$ ?

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b) There are 3 good and 1 bad coins. The bad one has head on both sides. A coin is chosen randomly and tosses 4 times. If head occurs all the 4 times what is the probability that the bad coin has been chosen for toss?
9) a) If the daily wage of 10000 workers in a factory follows normal distribution with mean and standard deviation of Rs. 70 and Rs. 5 respectively, find the expected number of workers whose daily wages are :
i) between Rs. 66 and Rs 72
ii) more than Rs. 72.

Here it is given that : $\frac{1}{\sqrt{2 \pi}} \int_{0}^{0} e^{-t^{2} / 2} \mathrm{~d} t=0.1554$ and
$\frac{1}{\sqrt{2 \pi}} \int_{0}^{0.8} e^{-t^{2} / 2} \mathrm{~d} t=0 \cdot 2881$ 7
b) Let $X$ denote the number of misprints on a page in a certain book. Assume that the random variable $X$ follows Poisson distribution. If $E\left(X^{2}\right)=6$ then find out the probability that a randomly chosen page will have at least one misprint.

