



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/M.Tech (ECE)/SEM-1/PGEC-101/2012-13**

**2012**

**ADVANCED ENGINEERING MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

Answer any *seven* questions.  $7 \times 10 = 70$

1. Let  $A$  be set of  $n$  elements and  $B$  be a set of  $m$  elements.
  - i) How many functions  $f: A \rightarrow B$  are possible ?
  - ii) How many one to one functions  $f: A \rightarrow B$ .
2.
  - a) There are 10 bulbs in a room each of which can be operated independently with 10 different switches. In how many ways the room can be illuminated ?
  - b) Find the minimum number of students in MCA first semester to be sure that at least six will receive the same grade, if there be five possible grades  $A, B, C, D$  and  $F$ .
3.
  - a) Define subgroup with example.
  - b) Let  $(G, 0)$  be a group. Prove that a non-empty subset  $H$  of  $G$  forms a subgroup of  $(G, 0)$  iff
    - i)  $a \in H, b$  not belongs to  $H \Rightarrow aob \in H$  and
    - ii)  $a \in H \Rightarrow a^{-1} \in H$ .



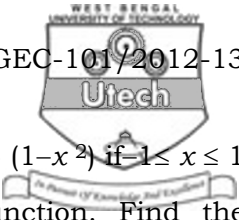
- c) Let  $(G, o)$  be a group. A non-empty subset  $H$  of  $G$  forms a subgroup of  $(G, o)$  if and only if
- $$a \in H, b \in H \Rightarrow aob^{-1} \in H. \text{ Prove it.}$$
4. a) i) Define cyclic group.  
ii) Prove that every cyclic group is Abelian.
- b) Show that a finite group  $(G, o)$  of order  $n$  is cyclic iff there exists an element  $b$  in  $G$  such that  $o(b) = n$ .
5. a) Prove that an integral domain  $D$  can be embedded in a field.
- b) Prove that a ring  $R$  can be embedded in a ring  $S$  with unity.
6. a) Prove that a tree with  $n$  vertices has  $n-1$  edges.
- b) Prove that any connected graph with  $n$  vertices and  $n-1$  edges is a tree.
7. Determine the inverse of matrix  $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$  using partition

method. Hence, find the solution of the system of equations

$$x_1 + x_2 + x_3 = 1$$

$$4x_1 + 3x_2 - x_3 = 6$$

$$3x_1 + 5x_2 + 3x_3 = 4$$



8. Let  $X$  have the density function  $f(x) = 0.75(1-x^2)$  if  $-1 \leq x \leq 1$  and otherwise. Find the distribution function. Find the probabilities  $P(-1/2 \leq X \leq 1/2)$  and  $P(1/4 \leq x \leq 2)$ .

Find  $x$  such that  $P(X \leq x) = 0.95$ .

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