Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/M.Tech (ECE)/SEM-1/PGEC-101/2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer any *seven* questions. $7 \times 10 = 70$

1. Let A be set of n elements and B be a set of m elements.

   i) How many functions $f : A \rightarrow B$ are possible ?

   ii) How many one to one functions $f : A \rightarrow B$.

2. a) There are 10 bulbs in a room each of which can be operated independently with 10 different switches. In how many ways the room can be illuminated ?

   b) Find the minimum number of students in MCA first semester to be sure that at least six will receive the same grade, if there be five possible grades A, B, C, D and F.

3. a) Define subgroup with example.

   b) Let (G,0) be a group. Prove that a non-empty subset H of G forms a subgroup of (G,0) iff

   i) $a \in H$, b not belongs to H = > $aob \in H$ and

   ii) $a \in H$ = > $a^{-1} \in H$.

c) Let (G,o) be a group. A non-empty subset H of G forms a subgroup of (G,o) if and only if

$a \in H,\ b \in H => aob^{-1} \in H$. Prove it.

4. a) i) Define cyclic group.

ii) Prove that every cyclic group is Abelian.

b) Show that a finite group (G,o) of order n is cyclic iff there exists an element b in G such that o(b) = n.

5. a) Prove that an integral domain D can be embedded in a field.

b) Prove that a ring R can be embedded in a ring S with unity.

6. a) Prove that a tree with n vertices has n–1 edges.

b) Prove that any connected graph with n vertices and n–1 edges is a tree.

7. Determine the inverse of matrix $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ using partition method. Hence, find the solution of the system of equations

$x_1 + x_2 + x_3 = 1$

$4x_1 + 3x_2 - x_3 = 6$

$3x_1 + 5x_2 + 3x_3 = 4$

8. Let X have the density function $f(x) = 0.75\,(1-x^2)$ if $-1 \le x \le 1$ and otherwise. Find the distribution function. Find the probabilities $P(-1/2 \le X \le 1/2)$ and $P(1/4 \le x \le 2)$.

   Find x such that $P(X \le x) = 0.95$.