Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS / M.TECH(ECE-M.COMM) / SEM-1 / MCE-101 / 2012-13

2012

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Attempt any *five* taking at least one from each Group. Each question carries equal marks.

GROUP - A

1. a) If the rate of motion $v = \frac{ds}{dt}$ is equal to x then the time t spent on translation along the curve $y = f(x)$ from one point $P_1(x1, y1)$ to another point $P_2(x2, y2)$ is a functional. Find the extremal of this functional, when $P_1(1, 0)$ and $P_2(2, 1)$.

   b) Suppose we have a refinery that must ship finished goods to some storage tanks. Suppose further that there are two pipelines, A and B, to do the shipping. The cost of shipping x units on A is ax^2; the cost of shipping y units on B is by^2, where $a > 0$ and $b > 0$ are given. How can we ship Q units while minimizing costs ? What happens to the cost if Q increases by $r\%$?

2. a) Minimize $(x - 2)^2 + 2 (y - 2)^2$

subject to

$x + 4y \leq 3$

$x \geq y$.

b) Using DPP, find the minimum value of $z = \sum_1^n y_i^2$ subject to $\sum_1^n y_i = c (\neq 0)$ and $y_i \geq 0$, $i = 1, 2, 3, \ldots\ldots\ldots\ldots, n$.

GROUP - B

3. a) Evaluate $\oint_c \frac{dz}{z^2 + 9}$ where

i) $c : | z - 3i | = 4$

ii) $c : | z + 3i | = 2$,

iii) $c : | z | = 5$.

b) What is the residue of $f (z) = \frac{1 + z}{1 - \cos z}$.

c) Evaluate $\oint_c \frac{z^2}{(z^2 + 3z + 2)^2}$ where

i) $c : | z | = 4$ and

ii) $c : | z - 1 | = \frac{3}{2}$.

4. a) Determine the Möbius transformation having 1 and i as fixed (invariant) points and maps 0 to – 1.

b) Find a Bilinear transformation which maps the upper half of the z-plane into the interior of a unit circle in the w-plane. verify the transformation.

GROUP - C

5. a) In a communication system the signal sent from point a to point b arrives by two paths in parallel. Over each path the signal passes through two repeaters (in series). Each repeater in one path has a probability of failing (becoming an open circuit) of 0.005. This probability is 0.008 for each repeater on the other path. All repeaters fail independently of each other. Find the probability that the signal will not arrive at point b.

   b) Two events A and B defined on a sample space S are related to a joint sample space through random variables X and Y and are defined by $A = \{ X \le x \}$ and $B = \{ y_1 \le Y \le y_2 \}$.

      i) Make a sketch of the two sample spaces showing areas corresponding to both events and the event $A \cap B = \{ X \le x, y_1 \le Y \le y_2 \}$.

      ii) Write a mathematical equation for joint distribution $F_{X,Y}(x, y)$.

      iii) Find and sketch the marginal distribution functions for the joint distribution function.

6. a) Two random variables X and Y are related by the expression $Y = aX + b$ where a and b are any real number.

      i) Show that their correlation coefficient is

      $$\rho = \begin{cases} 1 \text{ if } a > 0 \text{ for any } b \\ -1 \text{ if } a < 0 \text{ for any } b \end{cases}$$

      ii) Show that their covariance is $C_{XY} = a\,\sigma_x^2$ where σ_x^2 is the variance of X.

   b) Show that the correlation coefficient satisfies the expression

   $$|\rho| = \frac{|\mu_{11}|}{\sqrt{\mu_{02}\,\mu_{20}}} \le 1.$$

c) In a control system, a random voltage X is known to have a mean value $\overline{X} = m_1 = -2V$ and a second moment $\overline{X^2} = m_2 = 9V^2$. If the voltage X is amplified by an amplifier that gives an output $= -1.5X + 2$, find σ_x^2, $\overline{Y}$, $\overline{Y^2}$, σ_y^2, R_{XY} where symbols have their usual meaning.

GROUP - D

7. a) Find the largest Eigenvalue and the corresponding Eigenvector of the matrix

$$A = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$$

Find the error in the value of the largest Eigenvalue.

b) Evaluate the nth order determinant D_n given below, by forming the difference equation satisfied by it and solving it.

$$D_n = \begin{vmatrix} 1+x^2 & x & 0 & 0 & \dots & 0 \\ x & 1+x^2 & x & 0 & \dots & 0 \\ 0 & x & 1+x^2 & x & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix}$$

8. a) Given that $u(x, y)$ satisfies the equation $\nabla^2 u = 0$ and the boundary conditions $u(x, 0) = 0$, $u(x, 4) = 8 + 2x$, $u(0, y) = \frac{1}{2}y^2$ and $u(4, y) = y^2$, find the values of $u(i, j)$, $i = 1, 2, 3$; $j = 1, 2, 3$, correct to two places of decimals.

b) Discuss the stability of Runge-Kutta method of order four in solving the ordinary diffrential equation.