Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech(CT)/SEM-1/M(CT)-101/2010-11

2010-11

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* of the following.

1. a) Find the integral surface satisfying the *p.d.e.* $(x - y) p + (y - x - z) p = z$ and passing through the circle $x^2 + y^2 = 1, z = 1$

   b) Solve the name equation $\frac{\partial^2 \mu}{\partial t^2} = c^2 \frac{\partial^2 \mu}{\partial x^2}$ given that

$$\mu (0, t) = \mu (l, t) = 0$$

$$\mu (x, 0) = f (x)$$

and $\frac{\partial \mu}{\partial t} (x, 0) = 0 \qquad 0 < x < l$ 7 + 7

2. a) If A and B are two events, then using the axioms of the probability from that

$$P (A \cup B) = P (A) + P (B) - P (A \cap B) .$$

b) If A and B are independent events, then show that $\overline{A}$ and $\overline{B}$ are also independent.

c) Let X have a *p.d.f.* defined by

$$f(x) = kx(1-x) \qquad 0 \le x \le 4$$
$$= 0 \qquad \text{otherwise}$$

determine the constant k and the mean of $(1-2x)$.

4 + 4 + 6

3. a) Find the Fourier cosine transform of the function $f(x) = e^{-x}$ and level find

$$\int_0^{\infty} \frac{\text{cs}\,\lambda x}{1+\lambda^2}\, d\lambda$$

b) If Fourier transform of $f(x)$ is given by

$$F\{f(x)\} = F(s) = \int_0^{\infty} f(x)\, e^{isx}\, dx .$$

then prove that

i) $F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$ where $a \neq 0$

ii) $F\{f(x-a)\} = e^{isx} F(s)$ 6 + 4 + 4

4. a) Show that the iteration given by

$$x_{n+1} = \frac{1}{\sqrt{x_n + 1}}$$

converges to the root lying in $[0, 1]$ for the equation $x^3 + x^2 - 1 = 0$. Find out the value of above mentioned root correct upto 3 decimal places.

b) Discuss Newton-Raphson's method for determining a real simple root of the equation $f(x) = 0$ and use this to form an iteration formula to find out positive square root of a real positive number N.

Hence find out the value of $\sqrt{3}$ correct upto 2 decimal places. 6 + 8

5. a) Use Müller's method to find out a root of the equation $\cos x = x e^{x}$ in $[0, 1]$

(perform one iteration by choosing three initial approximations to the root as – 1, 0, 1).

b) Decompose the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

into LU form where L and U are lower triangular and upper triangular matrices respectively.

Hence, solve the system of equations

$$x_1 + 2x_2 + 3x_3 = 14$$

$$2x_1 + 5x_2 + 2x_3 = 18$$

$$3x_1 + x_2 + 5x_3 = 20.$$

6 + 8

6. a) State a set of sufficient conditions for convergence of Gauss-Siedel iteration for solving a set of linear equations and use this method to solve the following system correct upto 2-decimal places

$$54x + y + z = 110$$

$$2x + 15y + 6z = 72$$

$$-x + 6y + 27z = 85.$$

b) Solve $\frac{dy}{dx} = x(1+y)$ along with $y(1) = 1$ for finding out the value of $y(1\cdot1)$ by using Modified Ruler's method, correct upto two places of decimal (use step length $h = 0\cdot05$) 7 + 7

7. a) Find the value of $y(1\cdot1)$ by using 4th order Runge-Karfta method for the differential equation

$$\frac{dy}{dx} = \lambda - y \text{ with } y(1) = 1$$

(use the step length of $h = 0\cdot05$).

b) Use the method of steepest descent to minimise the function

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1 x_2 + x_2^2$$

starting from the point $x_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(perform one iteration).

c) Describe Monte Carlo approach for finding out approximate value of the integral

$$\int_a^b f(x)\,dx$$ 5 + 5 + 4