



Name :

Roll No. :

Invigilator's Signature :

**CS/M.Tech(CT)/SEM-1/M(CT)-101/2010-11
2010-11**

ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer any five of the following.

1. a) Find the integral surface satisfying the *p.d.e.*
 $(x - y)p + (y - x - z)p = z$ and passing through the
circle $x^2 + y^2 = 1, z = 1$

- b) Solve the name equation $\frac{\partial^2 \mu}{\partial t^2} = c^2 \frac{\partial^2 \mu}{\partial x^2}$ given that

$$\mu(0, t) = \mu(l, t) = 0$$

$$\mu(x, 0) = f(x)$$

$$\text{and } \frac{\partial \mu}{\partial t}(x, 0) = 0 \quad 0 < x < l \quad 7 + 7$$

2. a) If A and B are two events, then using the axioms of the
probability from that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$



- b) If A and B are independent events, then show that A and \overline{B} are also independent.

- c) Let X have a *p.d.f.* defined by

$$f(x) = kx(1-x) \quad 0 \leq x \leq 4$$

$$= 0 \quad \text{otherwise}$$

determine the constant k and the mean of $(1 - 2x)$.

4 + 4 + 6

3. a) Find the Fourier cosine transform of the function

$f(x) = e^{-x}$ and level find

$$\int_0^{\infty} \frac{\cos \lambda x}{1 + \lambda^2} d\lambda$$

- b) If Fourier transform of $f(x)$ is given by

$$F\{f(x)\} = F(s) = \int_0^{\infty} f(x) e^{isx} dx.$$

then prove that

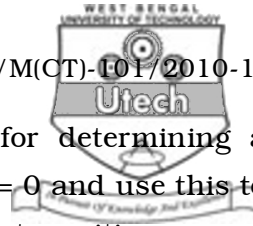
i) $F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$ where $a \neq 0$

ii) $F\{f(x-a)\} = e^{isx} F(s)$ 6 + 4 + 4

4. a) Show that the iteration given by

$$x_{n+1} = \frac{1}{\sqrt{x_n + 1}}$$

converges to the root lying in $[0, 1]$ for the equation $x^3 + x^2 - 1 = 0$. Find out the value of above mentioned root correct upto 3 decimal places.



- b) Discuss Newton-Raphson's method for determining a real simple root of the equation $f(x) = 0$ and use this to form an iteration formula to find out positive square root of a real positive number N .

Hence find out the value of $\sqrt{3}$ correct upto 2 decimal places. 6 + 8

5. a) Use Müller's method to find out a root of the equation $\cos x = x e^x$ in $[0, 1]$

(perform one iteration by choosing three initial approximations to the root as $-1, 0, 1$).

- b) Decompose the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

into LU form where L and U are lower triangular and upper triangular matrices respectively.

Hence, solve the system of equations

$$x_1 + 2x_2 + 3x_3 = 14$$

$$2x_1 + 5x_2 + 2x_3 = 18$$

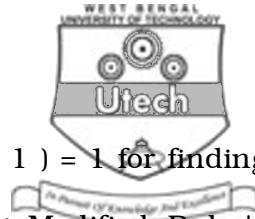
$$3x_1 + x_2 + 5x_3 = 20. \quad \text{6 + 8}$$

6. a) State a set of sufficient conditions for convergence of Gauss-Siedel iteration for solving a set of linear equations and use this method to solve the following system correct upto 2-decimal places

$$54x + y + z = 110$$

$$2x + 15y + 6z = 72$$

$$-x + 6y + 27z = 85.$$



- b) Solve $\frac{dy}{dx} = x(1+y)$ along with $y(1) = 1$ for finding out the value of $y(1.1)$ by using Modified Euler's method, correct upto two places of decimal (use step length $h = 0.05$) 7 + 7

7. a) Find the value of $y(1.1)$ by using 4th order Runge-Karfta method for the differential equation

$$\frac{dy}{dx} = \lambda - y \text{ with } y(1) = 1$$

(use the step length of $h = 0.05$).

- b) Use the method of steepest descent to minimise the function

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1 x_2 + x_2^2$$

starting from the point $x_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(perform one iteration).

- c) Describe Monte Carlo approach for finding out approximate value of the integral

$$\int_a^b f(x) dx \quad \text{5 + 5 + 4}$$

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