Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH(CSE)/SEM-3/CST-1133A/2012-13

2012

MATHEMATICAL MODELLING AND NUMERICAL SIMULATION

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) For a two-person game with *A* and *B*, the minimizing and the maximizing players, the optimum strategies are

a) Minimax for *A* and Maximin for *B*

b) Maximin for *A* and Minimax for *B*

c) Minimin for *A* and Maximin for *B*

d) None of these.

ii) In a fair game the value of the game is

a) 1 b) 0

c) Unbounded d) None of these.

iii) A hyper plane is a convex set.

a) True b) False.

iv) Lagrange interpolation formula is used

a) near the beginning of the table

b) near the end of the table

c) in the middle point of the table

d) at all there.

v) The error in Weddle formula is

a) $-\frac{h^7}{840}(b-a)f^5(x)$

b) $-\frac{h^6}{840}(b-a)f^6(x)$

c) $-\frac{h^4}{840}(b-a)f^4(x)$

d) none of these.

vi) Runge-Kutta method of 4th order is used

a) to interpolate

b) to solve a non-linear equation

c) to evaluate a definite integral

d) to solve differential equation.

vii) The equation $AX = B$ have multiple solution if

a) Rank A = Rank (A, B) $\neq$ number of unknown

b) Rank A = Rank (A, B) = number of unknown

c) Rank A < Rank (A, B)

d) Rank A > Rank (A, B).

viii) Deterministic models follows :

a) Kepler's laws

b) Stochastic processes

c) Both (a) and (b)

d) None of these.

ix) Which relation is true ?

a) $E = e^{hd}$

b) $E = e^{-hd}$

c) $D = \log E$

d) $E = \log D$.

x) Interpolation in regular interval at the end of the table is done using

a) Newton forward formula

b) Newton backward formula

c) Newton divided difference formula

d) Gauss forward formula.

xi) For unequal interval which of the following formula is not used ?

a) Newto - divided difference formula

b) Lagrange interpolation formula

c) NG Forward

d) Bessel interpolation formula.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If the transition matrix of a Markov chain is

$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

Find the stationary distribution.

3. Find if the Markov chain with transition

$$\begin{bmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 0 & 0 & 1 & 0 \\ 1/3 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{is}$$

irreducible, what can you say about state 3 ?

4. What is Mathematical Model ?

5. What is the application of Markov process ?

6. Define Random and non-random models.

7. Find the unique fixed probability vector of the regular stochastic matrix

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1\backslash 2 & 1\backslash 2 & 0 \end{pmatrix}$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) For a transition matrix

$\begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix}$, find its steady state probability distribution.

b) What is Stochastic model ?

9. Find the nodal values of the wave equation $\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$.

Given that $u(0, t) = 0$, $u(5, t) = 0$, $u(x, 0) = x^2(5 - x)$ and $u_1(x, 0) = 0$. Take $h = 1$ and up to one half of the period of vibration.

10. a) Use the Runge-Kutta method to solve

$$10 \frac{dy}{dx} = x^2 + y^2, \; y(0) = 1$$

For the interval $0 < x \leq 0.4$ with $h = 0.1$.

b) A manufacturing company has a certain piece of equipment that is inspected at the of each day and classified as just overhauled, good, fair or imperative. If the item is imperative it is overhauled, the procedure that takes one day. Let us denote the four classification as states 1, 2, 3 and 4 respectively. Assume that the working condition of the equipment follows a Markov process with the following transition matrix.

$$P = \begin{bmatrix} 0 & 3/4 & 1/4 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

If costs Rs. 125 to overhaul a machine on the average and Rs. 75 if production is lost if a machine is found imperative, using the steady-state probabilities, compute the expected per day cost of maintenance.

11. a) Suppose there are two market products A and B. Let each of the brands have exactly 50% of the total market in the same period and the market size is fixed. The transition matrix is given below :

$$\begin{pmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{pmatrix}$$

If the initial market share break down is 50% for each brand, then determine their market shares in the steady state.

b) Define air pollution model.