Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/M.TECH(CSE)/SEM-2/CSEM-405/2013

2013

OPERATIONS RESEARCH & OPTIMIZATION TECHNIQUES

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph Sheet(s) will be provided by the Institution on demand.

Answer Q. No. **1** and any *four* from the rest. $5 \times 14 = 70$

1. Answer the following questions briefly, giving reasons, explanations and examples :

   a) An Integer Programming (IP) problem *P* has a bounded feasible region. Is it possible for *P*'s optimal solution to lie in the interior of the feasible region and not on the perimeter ? 4

b) How many Corner Point Feasible (CPF) solutions does the following Linear Programming (LP) problem have ?

Minimize $Z = 2x_1 + x_2$ 5

subject to $2x_1 + 3x_2 \geq 6$

$3x_1 + 4x_2 \leq 12$

$4x_1 + 5x_2 > 16$

and $x_1 \geq 0,\ x_2 \geq 0$.

c) Determine the optimal solution to the Mathematical Programming problem given below. Here the objective function Z has two different forms in two different parts of the feasible region.

Maximize $Z = 2x_1 + x_2$, when $x_1 + x_2 \leq 8$

$Z = x_1 + 2x_2$, when $x_1 + x_2 > 8$

subject to $x_1 + x_2 \leq 10$

$2x_1 + x_2 \leq 16$

and $x_1 \geq 0, x_2 \geq 0$. 5

2. a) Consider the following Linear Programming (LP) problem :

Maximize $Z = 5x_1 + 4x_2$

subject to $4x_1 + 3x_2 \le 24$

$x_1 + x_2 \le 7$

$x_1 + 2x_2 \le 8$

and $x_1 \ge 0, x_2 \ge 0$.

Determine the optimal solution to this problem using the graphical method. Clearly indicate the feasible region and list all corner point feasible (CPF) solutions. 7

b) The above LP problem has been changed as follows :

Maximize $Z = 4x_1 + 5x_2$

subject to $4x_1 + 3x_2 = 24$

$x_1 + x_2 \ge 6$

$x_1 + 2x_2 \le 12$

and $x_1 \ge 0, \; x_2 \ge 0$.

Again, determine the optimal solution using the graphical method. As before, clearly indicate the feasible region and list the CPF solutions. 7

3. Solve the following LP problem algebraically using the simplex tabular format :

Maximize $Z = 5x_1 - 3x_2 + 3x_3$

subject to $-6x_1 + 2x_2 + 4x_3 \le 12$

$2x_1 + 5x_2 + x_3 \le 10$

$x_2 - x_3 \le 4$

and $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$.

4. The following terms and concepts have special meaning and significance in the context of Mathematical Programming. Briefly explain each of the terms and concepts with the help of suitable examples : 4 + 5 + 5

a) Concave function

b) Shadow price

c) Branch-and-bound method with LP relaxation.

5. Bharati Shoe Co. (BSC) can sell up to 200 pairs of its Model A shoes and 100 pairs of its Model B shoes per month. The production process involves three departments : Design (DS), Fabrication (FB) and Finishing & Packaging (FP), as indicated below :

Dept	Monthly hrs. available	Hrs. per pair (Model A)	Hrs. per pair (Model B)
DS	2,000	15	10
FB	4,200	20	25
FP	2,500	10	15
Manufacturing cost per pair		Rs. 1,350 (Model A)	Rs. 1,500 (Model B)
Selling price per pair		Rs. 1,600 (Model A)	Rs. 1,800 (Model B)

BSC also has the option of obtaining additional pairs of shoes from a subconductor, who has offered to supply to BSC up to a total of 150 pairs per month in any combination of Models A and B at a uniform cost of Rs. 1,400 per pair. How many pairs of each type should BSC make in its factory and how many should it buy from the subcontractor in order to maximize its profit ?

Provide an Integer Programming (IP) formulation of the problem, explaining each step carefully. (You do not need to solve the problem).

6. a) Explain how dynamic programming can be employed to parenthesize a matrix chain product and thereby minimize the number of scalar multiplications. How does this reduce the time needed to compute the product ? 7

   b) In a matrix chain product, the sequence of dimensions is (3, 4, 3, 5, 4, 6, 4, 3). What is the best way to parenthesize this product and minimize the total computation time ? 7

7. A construction project has twelve activities. For each activity, its duration (in weeks), its immediate predecessor activity (or activities), and the minimum number of labourers needed for the activity, are shown below :

Activity	Duration (wks)	Immediate Predecessor activities	Minimum number of labourers needed
A	6	—	30
B	7	*A*	70
C	5	*B*	65
D	8	*B*	55
E	3	*C, D*	110

Contd..

Activity	Duration (wks)	Immediate Predecessor activities	Minimum number of labourers needed
F	2	*C, E*	100
G	5	*C, F*	60
H	6	*E*	90
K	9	*G, H*	75
L	5	*G, K*	80
M	8	*E, F*	40
N	6	*M, L*	35

a) Determine the minimum duration of this project in weeks. Given that a total of 160 labourers are available. 7

b) Would it be possible to complete the project in a shorter time if 200 labourers are made available ? 7

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