Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/M.TECH(CSE)/SEM-2/CSEM-405/2012

2012
OPERATIONS RESEARCH & OPTIMIZATION TECHNIQUES

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer question No. 1 and any *four* questions from the rest.

1. Answer each of the following questions briefly. 5 + 5 + 4

a) A Linear Programming (LP) problem has two decision variables and six feasibility constraints. Both the decision variables satisfy non-negativity constraints and the feasible region is bounded. Is it possible in this case for the feasible region to have more than eight cornet points (*i.e.*, CPF solutions) ? Give reasons for your answer.

b) How will you solve the following Non-Linear Programming (NLP) problem ? (You can choose any method you prefer). Show the solution steps in detail.

Minimize $Z = 2x_1 + x_2$

subject to $x_1^2 + 4x_2^2 \geq 4$

and $x_1 \geq 0, x_2 \geq 0$

c) In which situations do we need to make use of the big-M method when solving Linear Programming (LP) and Integer Programming (IP) problems ? Explain with examples.

2. a) Kunal Engineering Company (KEC) realizes that it can significantly increase its profits by starting the manufacture of two new products *A* and *B*. But it would like to begin by producing just one of them at this time. Both products must be made in whole units, *i.e.*, the number of units produced must be an integer. KEC has three factories *P*, *Q* and *R*, and each of them can manufacture both products, but the profits per unit differ slightly. The chosen product can be made at one or more factories. The following data is supplied :

	Production time per unit (hrs)		Production time available (hrs)	Profit per unit (Rs. thousands)	
	Product A	Product B		Product A	Product B
Factory P	6	8	1,200	6,200	7,000
Factory Q	7	10	1,500	6,500	7,200
Factory R	6.5	9.5	1,300	6,000	7,500

Formulate this problem as an Integer Programming (IP) problem assuming that the profit must be maximized and only one of the two products will be manufactured. Explain your formulation in words. (You need not solve the problem).

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b) Consider the problem described above in part (a). Suppose KEC now decides to manufacture both the products *A* and *B*. Also suppose that the products can be manufactured in fractions of a unit. Reformulate the problem as a Linear Programming (LP) problem assuming the profit must be mamimized. Explain your formulation in words. (You need not solve the problem.) 6

3. a) Consider the following Linear Programming (LP) problem: 7

Minimize $Z = 3x_1 + 8x_2$

subject to

$$3x_1 + 2x_2 \geq 12$$
$$x_2 \leq 4$$
$$x_1 + x_2 \leq 5$$
and $x_1 \geq 0, x_2 \geq 0$

Determine the optimal solution to this problem using the graphical method. Clearly indicate the feasible region and list all corner point feasible (CPF) solutions.

b) The above LP problem has been changed as follows : 7

Maximize $Z = 3x_1 + 8x_2$

subject to

$$3x_1 + 2x_2 = 12$$
$$x_2 \geq 2$$
$$x_1 + x_2 \leq 5$$
and $x_1 \geq 0, x_2 \geq 0$

Again determine the optimal solution using the graphical method. As before, clearly indicate the feasible region and list all corner point feasible (CPF) solutions.

4. Solve the following LP problem algebraically using the simplex tabular format : 14

Maximize $Z = 4x_1 - 2x_2 + x_3$

subject to

$-5x_1 + x_2 + 3x_3 \leq 10$

$x_1 + 4x_2 + x_3 \leq 8$

$2x_1 - 2x_3 \leq 7$

and $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

5. The following terms and concepts have special meaning and significance in the context of LP and the simplex method. Briefly explain each of the terms and concepts with the help of suitable examples : 3 + 4 + 3 + 4

a) Artificial variable ;

b) Surplus variable ;

c) Decision variables with negative values ;

d) Sensitivity analysis.

6. A construction project has ten activities. For each activity, its duration (in weeks), its immediate predecessor activity (or activities), and the minimum number of workers needed for the activity, are shown below :

Activity	Duration (wks)	Immediate predecessor activities	Minimum number of workers needed
A	6	–	30
B	7	A	70
C	5	B	65
D	8	A	55
E	3	B,D	110
F	2	E	100
G	5	C,F	60
H	6	E	90
K	9	G,H	75
L	5	G,K	80

a) For this project, identify the critical path and determine its length in weeks. 7

b) Determine whether it would be possible to complete the project 3 + 4

i) in 38 weeks with a total of 190 workers ;

ii) in 43 weeks with a total of 150 workers.

7. Nibedita Pharmaceuticals (NP), a medical firm, has divided the city of Kolkata into three zones (Zone A, Zone B, Zone C). NP has a total of nine sales representatives (SRs) who can be assigned for duty to the three zones. there is no restriction on how many SRs can be allocated to a particular zone ; it can be any number between zero and nine. But of course the total number allocated to all the zones taken together must be nine. The table shown below gives the total expected sales when different numbers of SRs are allocated to each zone. Note that a larger number of SRs when assigned to a zone can sometimes cause a decrease in sales because of various unforeseen reasons.

a) Use dynamic programming to determine how many SRs should be assigned to each zone to maximize the total expected sales. 9

b) In order to increase sales in Zone 2, NP has subsequently decided to allocate four SRs to that zone.

How should the remaining five SRs be allocated to Zone 1 and Zone 3 to maximize the total expected sales ? 5

Total Expected Sales (Rs. thousands)

Number of SRs	zone A	Zone B	Zone C
0	30	35	42
1	45	45	54
2	60	52	60
3	70	64	70
4	79	72	82
5	90	82	95
6	98	93	102
7	105	98	110
8	100	100	110
9	90	100	110