Name : ....

Roll No. : ....

Invigilator's Signature : ....

2011

INFORMATION AND CODING THEORY

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *seven* questions.   7 × 10 = 70

1. Two zero memory sources S_1 and S_2 have q_1 and q_2 symbols, respectively. The symbols of S_1 occur with probabilities P_i, $i = 1, 2, ... q_1$; the symbols of S_2 occur with probabilities Q_i, $i = 1, 2, ... q_2$ and the entropies of S_1 and S_2 are H_1 and H_2 respectively. A new zero-memory source S (y), called a mixture of S_1 and S_2, is formed with $q_1 + q_2$ symbols. The first q_1 symbols of S (y) have probabilities yP_i, $i = 1, 2, ... q_1$, and the last q_2 symbols of S (y) have probabilities $y'Q_i$, $i = 1, 2, ... q_2$ ($y' = y - 1$).

   a) Show that $H\left[S(y)\right] = yH_1 + y' H_2 + H(y)$.

   b) Express y_0, the value of y which maximizes H [S (y)], in terms of H_1 and H_2. Find $H\left[S(y_0)\right]$.   6 + 4

2. A source has six possible outputs with probabilities as shown in the table below. Codes 1, 2, 3, 4, 5 and 6, as given in the table are considered.

Output	$P(s_i)$	**1**	**2**	**3**	**4**	**5**	**6**
S_1	$\frac{1}{2}$	000	0	0	0	0	0
S_2	$\frac{1}{4}$	001	01	10	10	10	100
S_3	$\frac{1}{16}$	010	011	110	110	1100	101
S_4	$\frac{1}{16}$	011	0111	1110	1110	1101	110
S_5	$\frac{1}{16}$	100	01111	11110	1011	1110	111
S_6	$\frac{1}{16}$	101	011111	111110	1101	1111	001

a) Which of these codes are uniquely decodable ?

b) Which are instantaneous ?

c) Find the average length L for all uniquely decodable codes. 3 + 3 + 4

3. According to the table given below :

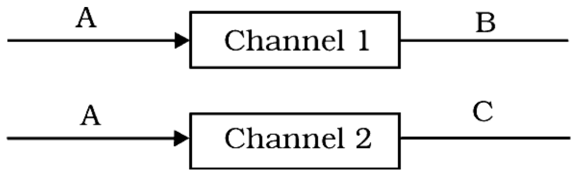
a) Find $H(S)$ and $H_3(S)$.

b) Find a compact code $H(S)$ when $X = \{0, 1\}$ and $X = \{0, 1, 2\}$.

c) Compute L for both the above codes : 3 + 4 + 3

S	S_1	S_2	S_3	S_4	S_5	S_6	S_7
$P(s_i)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$

4.



Each time an input symbol is transmitted over channel 1. It is repeated over channel 2 (see figure), so that may be considered to be a pair of symbols $\left(b_j,\ c_k\right)$. Furthermore we assume that the repetition is performed independently of the results of the original transmission, so that $P\left(c_k / a_i, b_j\right) = P\left(c_k,\ a_i\right)$. Note that this does not mean c_k and b_j are statistically independent, $P\left(c_k / b_j\right)$ not equal to $P\left(c_k\right)$.

a) Show that $I\ (\ A;\ B;\ C\ ) = I\ (\ A;\ B\ ) + I\ (\ A;\ C\ ) - I\ (\ B;\ C\ )$.

b) Generalize part a to n channels. 6 + 4

5. A binary multiplicative channel has 2 binary inputs and one binary output, $b = ac$. This channel may be described as an ordinary zero-memory channel by considering four possible output combinations to compromise a new input alphabet A' :

$A' = (\ 00, 01, 10, 11\ \}$

a) Write the channel matrix for the channel with input A' and output B.

b) The input symbols a and c are selected independently. $P_r\left\{a = 0\right\} = w_1$, $P_r\left\{c = 0\right\} = w_2$. Define $1 - w_1 = w_1'$ and $1 - w_2 = w_2'$. Find $I\ (\ A' : B\ )$. 6 + 4

6. A uniform channel has r inputs. These inputs are chosen with equal probabilities, and it is found that the maximum likelihood decision procedure produces a probability of error p. Find a lower bound to the equivocation $H(A/B)$ in terms r or p, or both. The lower bound of 0 is not acceptable.

7. The source S has 9 symbols each occurs with probability 1/9.

   a) Find a compact code using the code alphabet $X = \{0\ 1\}$

   b) Find a compact code using the code alphabet $X = \{0, 1, 2\}$

   c) Find a compact code using the code alphabet $X = \{0, 1, 2, 3\}$. 3 + 3 + 4

8. A zero memory binary source has $P(0) = 0{\cdot}1$ and $P(1) = 0{\cdot}9$.

   a) Find $H(S)$.

   b) Find L.

   c) Find $\frac{L_n}{n}$ for $n = 2, 3, 4$ and $n \to \propto$, when S^n is encoded into a compact code, still with $X = \{0, 1\}$.

   d) Find the efficiency of you four codes. $4 \times 2\frac{1}{2}$

9. Answer the following :

   a) Consider the third order Markov source where the probability of emitting a 0 or a 1 does not depend upon the previous two symbols but does depend on the third symbol back. The probability that the next symbol will be the same as the third symbol back is 0·9. Draw the state diagram for this source.

   b) Find the entropy of this source. 6 + 4