



Name :

Roll No. :

Invigilator's Signature :

CS/M.Tech(CST)/SEM-1/CST-1103A2/2010-11

2010-11

LOGIC & LOGIC PROGRAMMING

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Objective Type Questions)

1. State whether the following statements are True or False :

$$10 \times 1 = 10$$

- i) Sentential symbols can even take real values between 0 and 1.
- ii) A set of formulas which aren't satisfiable by all truth assignments can tautologically imply any formula.
- iii) \neg and \wedge form a complete set, i.e., any Boolean function can be realized by them.
- iv) If $\forall x \varphi(x)$ is true, then for a minority of values of x , $\varphi(x)$ is true.
- v) If $\exists x \varphi(x)$ is true, then for at least one value of x , $\varphi(x)$ is true.
- vi) De-Morgan's laws are tautologies.



- vii) Functions in first order logic need not take values 0 or 1.
- viii) In deductive calculus we only use proof by induction.
- ix) Compactness theorem concludes about satisfiability of an infinite set from its finite subsets.
- x) Every first order logic sentence can also be represented by sentential logic.

GROUP – B

(Short Answer Type Questions)

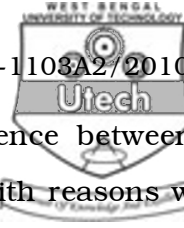
Answer any *three* of the following. $3 \times 5 = 15$

2. a) Define well-formed-formulas.
- b) Check whether following two formulas are wff by checking their ancestral tree :

$$((A_1 \wedge A_{10}) \rightarrow ((\neg A_3) \vee (A_8 \times A_3)))$$

$$((A_2 \vee A_5) \times A_3 \vee A_4 \wedge A_6)$$

3. Write down truth tables of $(A \rightarrow B)$ and $(A \times B)$ and explain why they are so.
4. Convert the following sentences to first order logic wff :
 - a) Anything anyone eats and isn't killed by is a food.
 - b) Hari can't do any job right.
 - c) $\epsilon - \delta$ definition of limit of a function.
 - d) $2^3 + 1^2 = 9$.



5. What is a structure ? What is the difference between a structure and a model ? Give two models with reasons why they are so for the sentence $\exists x \forall y (\neg y \supset x)$.
6. Write prolog programs for the following problems and explain with data how they work :
- GCD of two numbers
 - Factorial of a number.

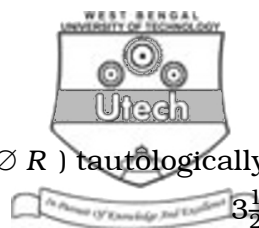
GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Prove that if S is a set of wffs containing all the sentence symbols from which wffs are to be build and closed under all five formula building operations, then S is a set of all wffs on those sentential symbols. 6
- b) Show that
- $$((A_2 \supset (A_1 \supset A_6)) \supset ((A_2 \wedge A_1) \supset A_6)) \text{ is a tautology.} \quad 3\frac{1}{2}$$
- c) Show that the following two formulas doesn't tautologically imply the other :

$$(A \times (B \times C)), ((A \wedge (B \wedge C)) \vee ((\neg A) \wedge (\neg B) \wedge (\neg C))). \quad 2$$



- d) Determine whether or not $(P \wedge Q) \supset R$ tautologically implies $((P \supset R) \vee (Q \supset R))$. 3½
8. a) Let $B \subseteq U$ and a class of functions $f : U \rightarrow U$ and $g : U \rightarrow U$ operate on members of U . Explain what is an inductive set S in U , define C^* , briefly argue why C^* is inductive. Define C_* , 6
- b) Suppose $B = \{a, b, c\}$ and C is generated from B by binary operation f and unary operation g . List all the members of C_2 . How many members might C_3 have? 4
- c) Prove $C^* = C_*$. 5
9. a) Define freely generated set. Set of natural numbers and set of integers which is freely generated and which is not explain. 4
- b) Give a proof of the fact that the set of wffs is freely generated. 7
- c) What is meant by a valid formula in first order logic? Show that θ is valid if and only if $\forall x\theta$ is valid. 4
10. a) Find the clausal form of the following wff :
 $\exists x \forall y (\forall z P(f(x), y, z) \supset (\exists u Q(x, u) \wedge \exists x R(y, v)))$. 3
- b) i) Explain the inference rule modus ponens.
 ii) What is a deduction of φ from a set of formulas Γ ?
 iii) Write down the forms of logical axioms. 6



- c) Prove deduction theorem, *i.e.*, if $\Gamma ; \gamma \vdash \varphi$ then
 $\Gamma \vdash \gamma \Rightarrow \varphi$ and the contraposition theorem, *i.e.*, if
 $\Gamma ; \varphi \vdash \neg \psi$ then $\Gamma ; \psi \vdash \neg \varphi$.

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