Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.Tech (CSE)/SEM-1/CS-905/2009-10

2009

INFORMATION AND CODING THEORY

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five* questions. $5 \times 14 = 70$

1. a) Define code rate in block code. Discuss the role of redundant words in odd and even parity block code. 4

   b) An (n, $n - 1$) single parity check code is used for error detection in a channel with bit error probability 10^{-3}. Find the maximum block length n such that success rate does not follow below 99%. 5

   c) The (8, 7) single parity check code with even parity is given. Find the probability of correct decoding, a decoding error and decoding failure when $p = 0 \cdot 001$. 5

2. a) State the condition when the code is linear. Show that any two code words from (4, 3) odd parity code fails to give a code word where the two code words from (4, 3) even parity code succeeds to give a code word. 2 + 4

b) Show that the generator matrix G and parity check matrix H of the (6, 3) code satisfy $GH^T = 0$. 3

c) The code words C have been furnished in table below :

i	C
000	000000
001	001110
010	010101
011	011011
100	100011
101	101101
110	110110
111	111000

Show that $CH^T = 0$. 5

3. a) Define cyclic code with examples. 2

b) Given the (7, 4) cyclic code with $g(x) = x^3 + x + 1$, determine systematic and non-systematic code word polynomials for the information polynomial $i(x) = x^3 + x^2 + x + 1$. Show that the systematic code word can be obtained using the quotient of $x^3\, i(x) \div g(x)$ instead of the remainder. 7

c) Show that $x^4 + x^2 + x + 1 = 0$ modulo ($x + 1$) 5

4. a) Differentiate additive inverse and multiplicative inverse of any number under modulo 7 prime field. 3

b) Show that modulo 8 multiplication over { 1, 2, 3, 4, 5, 6, 7 } fails to form a multiplicative group. 5

c) Show that the row vectors of the matrix :

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

are linearly independent. 6

5. a) Explain the generation of Galois field $GF\left(2^4\right)$ defining the field element α. 3

b) Show that $\alpha^{15} = 1$. 4

c) Find the determinant of the matrix $A = \begin{bmatrix} 1 & \alpha^4 & \alpha^3 \\ \alpha^2 & 0 & \alpha \\ \alpha^4 & \alpha & \alpha^5 \end{bmatrix}$ over $GF\left(2^3\right)$ and $GF\left(2^4\right)$. 7

6. a) Explain how BCH (Bose-Choudhuri-Hocquenghem) Code has been constructed. 3

b) Show that the triple error correcting binary BCH code constructed over $GF(2^4)$ has generator polynomial $g(x) = x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1$. 5

c) Show that $\alpha, \alpha^2, \alpha^3$ and α^4 are roots of $g(x) = x^8 + x^7 + x^6 + x^4 + 1$, the generator polynomial of the (15, 7) double error-correcting binary BCH code, where α is a primitive field element of $GF(2^4)$. 6

7. Write short notes on any *two* of the following : 2×7

a) Reed-Solomon code over BCH code

b) Role of Hamming code in using code word

c) Function of syndrome tackle in error correction

d) Use of digital channel for error control coding.