Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/ME (CSE)/SEM-1/PGCSE-101/2009-10

2010

GRAPH THEORY & COMBINATORICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be provided by the Institution.

GROUP – A
(Objective Type Questions)

1. Answer any *five* questions : $5 \times 2 = 10$

   i) Write the incidence matrix of the directed graph :

   ii) Under what conditions does a graph contain an Euler trail ?

iii) If L_4 denotes a path having 4 vertices, then what is its chromatic polynomial ? Justify your answer.

iv) Draw the ring sum of the following graphs :

v) Prove that $C(n-1, r) + C(n-1, r-1) = C(n, r)$

vi) Find the recurrence relation for the closed form expression $u_n = 10 \cdot 5^n$.

vii) Using multinomial theorem, find the coefficient of $x^2 y^3 z^2$ in the expansion of $(x+y+z)^7$.

viii) Define stirling number of the second kind.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. State and prove multinomial theorem.

3. Applying Vandermonde's identity, prove that $C(3n, 3) = 3C(n, 3) + 6nC(n, 2) + n^3$.

4. Examine whether the following graphs are isomorphic :

5. Find the chromatic polynomial of the following graph :

6. Prove that a graph is a tree if and only if it is minimally connected.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) i) Define Bell number.

ii) Prove that $S(n, n-1) = C(n, 2)$ for $n \geq 2$ where S is the stirling number of the 2nd kind. 2 + 3

b) Give the combinatorial proof of the identity

$$C(n, 0) + C(n, 1) + C(n, 2) + \ldots.. + C(n, n) = 2^n$$

[Binomial theorem must not be used] 5

c) A, B and C are three towns. There are 5 different roads from A to B, different roads from B to C and 3 different roads from A to C. How many different ways are there to go from A to C and back again to A that visit B both while going and coming ? 5

8. a) Prove the derangement formula :

$$D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots.. + (-1)^n \cdot \frac{1}{n!}\right)$$ 10

b) Find the number of integers between 1 and 1000 inclusive that are divisible by none of 5, 6 and 8. 5

9. a) Solve the recurrence relation :

$$u_n - 2u_{n-1} = 6_n , u_1 = 2$$ 5

b) Find a minimal spanning tree of the following graph by Kruskal's algorithm : 5

c) Draw if possible, the graphs having the following properties :

i) a simple graph with degree sequence (1, 2, 2, 4, 4)

ii) a graph, not necessarily simple, having degree-sequence (1, 2, 2, 4, 5).

[Give reasons if not possible.] 2 + 3

10. a) State and prove Euler's theorem on planar graphs. 10

b) Five citizens c_1, c_2, c_3, c_4, c_5 are members of three associations a_1, a_2, a_3 as follows :

c_1 is a member of a_1 & a_2, c_2 of a_1, c_3 of a_2 & a_3, c_4 of a_2 & a_3, c_5 of a_3. Examine whether it is possible to from a committee of 3 citizens in such a way that the committee has representatives from all the three associations. 5

11. a) Find the maximum flow through the following network by Ford-Fulkerson's algorithm : 12

b) What do you mean by a k-connected graph and a separable graph ? 3