

Name :

Roll No. :

Invigilator's Signature :

CS/ME (CSE)/SEM-1/PGCSE-101/2009-10

2010

GRAPH THEORY & COMBINATORICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Graph sheet(s) will be provided by the Institution.

GROUP – A
(Objective Type Questions)

1. Answer any *five* questions : $5 \times 2 = 10$

i) Write the incidence matrix of the directed graph :

ii) Under what conditions does a graph contain an Euler trail ?



iii) If L_4 denotes a path having 4 vertices, then what is its chromatic polynomial ? Justify your answer.

iv) Draw the ring sum of the following graphs :

v) Prove that $C(n-1, r) + C(n-1, r-1) = C(n, r)$

vi) Find the recurrence relation for the closed form expression $u_n = 10 \cdot 5^n$.

vii) Using multinomial theorem, find the coefficient of $x^2 y^3 z^2$ in the expansion of $(x+y+z)^7$.

viii) Define stirling number of the second kind.

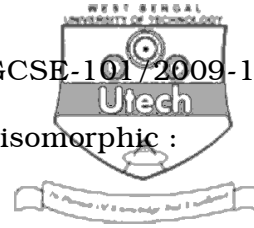
GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. State and prove multinomial theorem.

3. Applying Vandermonde's identity, prove that

$$C(3n, 3) = 3C(n, 3) + 6nC(n, 2) + n^3.$$



4. Examine whether the following graphs are isomorphic :

5. Find the chromatic polynomial of the following graph :

6. Prove that a graph is a tree if and only if it is minimally connected.

GROUP – C

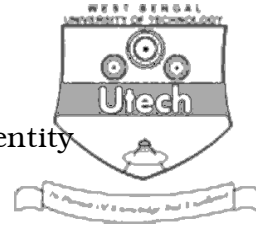
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) i) Define Bell number.

ii) Prove that $S(n, n-1) = C(n, 2)$ for $n \geq 2$ where

S is the stirling number of the 2nd kind. $2 + 3$



- b) Give the combinatorial proof of the identity

$$C(n, 0) + C(n, 1) + C(n, 2) + \dots + C(n, n) = 2^n$$

[Binomial theorem must not be used]

5

- c) A , B and C are three towns. There are 5 different roads from A to B , different roads from B to C and 3 different roads from A to C . How many different ways are there to go from A to C and back again to A that visit B both while going and coming ?

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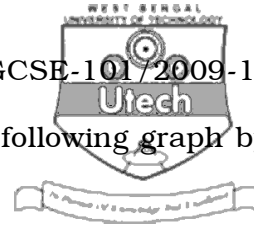
8. a) Prove the derangement formula :

$$D_n = n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \cdot \frac{1}{n!} \right) \quad 10$$

- b) Find the number of integers between 1 and 1000 inclusive that are divisible by none of 5, 6 and 8. 5

9. a) Solve the recurrence relation :

$$u_n - 2u_{n-1} = 6_n, u_1 = 2 \quad 5$$



- b) Find a minimal spanning tree of the following graph by

Kruskal's algorithm :

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- c) Draw if possible, the graphs having the following

properties :

- i) a simple graph with degree sequence

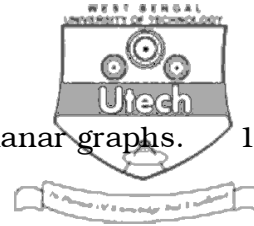
(1, 2, 2, 4, 4)

- ii) a graph, not necessarily simple, having degree-

sequence (1, 2, 2, 4, 5).

[Give reasons if not possible.]

2 + 3



10. a) State and prove Euler's theorem on planar graphs. 10

b) Five citizens c_1, c_2, c_3, c_4, c_5 are members of three associations a_1, a_2, a_3 as follows :

c_1 is a member of a_1 & a_2 , c_2 of a_1 , c_3 of a_2 & a_3 ,

c_4 of a_2 & a_3 , c_5 of a_3 . Examine whether it is possible

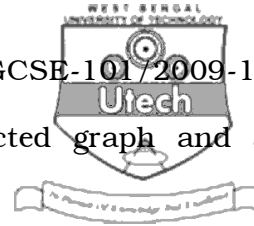
to form a committee of 3 citizens in such a way that the

committee has representatives from all the three

associations. 5

11. a) Find the maximum flow through the following network

by Ford-Fulkerson's algorithm : 12



- b) What do you mean by a k -connected graph and a separable graph ?

3

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