Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech (CSE)/SEM-1/CS-908/2010-11

2010-11

FUZZY SETS AND FUZZY LOGIC

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer in short, any *five* of the following : 5 × 2 = 10

a) What is lattice of fuzzy number ? Give example of it.

b) Calculate the following :

i) [– 1, 2] + [1, 3]

ii) [– 3, 4] × [– 3, 4]

c) Explain why law of contradiction and law of exclusive middle are violated in fuzzy set theory under standard fuzzy sets operation ? What is the significance of this ?

d) What are fuzzy t-norms and t-conorms ? Give an example of each ?

e) Give the arithmetical ($\times$, $\div$) interval opertion for two intervals I_1 (a, b) and I_2 (c, d).

f) What are fuzzy tautologies ? Give an example of it.

g) What is fuzzy ignorance ? Give an example of it.

h) Distinguish between chance and fuzziness, with proper justification.

2. a) Explain the vertex method for continuous value membership function and give its algorithm. 7

b) Let A and B be two fuzzy numbers given by : 8

$$\underline{A}(x) = \begin{cases} (x+2)/2 & \text{for } -2 < x \le 0 \\ (2-x)/2 & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\underline{B}(x) = \begin{cases} (x-2)/2 & \text{for } 2 < x \le 4 \\ (6-x)/2 & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the fuzzy numbers $\underline{A}+\underline{B}, \underline{A}-\underline{B}, \underline{B}-\underline{A}$ and $\underline{A}/\underline{B}$.

3. a) A "power supply" is required to convert 120 V AC to a useful voltage + 5 DC. Some power supply designs employ a technique called "switching", to generate the appropriate voltages. Consider two linguistic variables, "high" and "medium", on the voltage range of 0 to 200 V AC. 8

"high" = $\left\{\frac{0}{0}+\frac{0}{25}+\frac{0\cdot01}{50}+\frac{0\cdot1}{75}+\frac{0\cdot3}{100}+\frac{0\cdot6}{125}+\frac{0\cdot7}{150}+\frac{0\cdot9}{175}+\frac{1}{200}\right\}$

"medium" = $\left\{\frac{0\cdot3}{0}+\frac{0\cdot5}{25}+\frac{0\cdot6}{50}+\frac{0\cdot8}{75}+\frac{1}{100}+\frac{0\cdot9}{125}+\frac{0\cdot7}{150}+\frac{0\cdot3}{175}+\frac{0\cdot1}{200}\right\}$

Find :

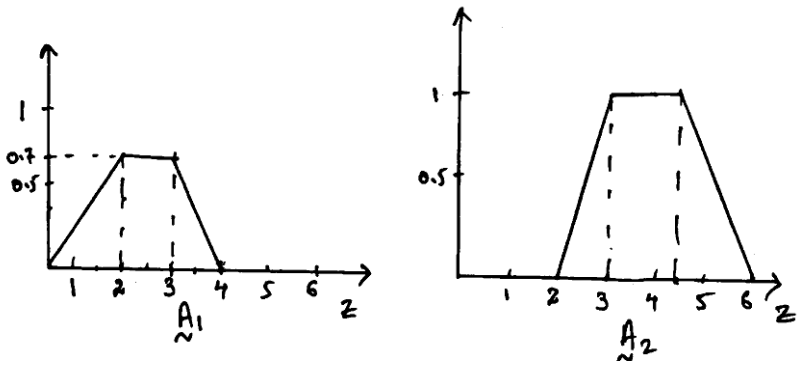
i) Not very high

ii) Slightly medium and very high

iii) Very, very high or very, very medium.

b) Explain Fuzzy *c*-Means (FCM). 7

4. a) Two companies bid for a contract and a committee has been constituted to review the estimates. The reviewed reports are evaluated on a non-dimensional scale and assigned a weighted score that is represented by a fuzzy membership function, as illustrated by the two fuzzy sets $\underline{A}_1$ and $\underline{A}_2$ in the figure below. The lowest bid has to be found out, as well as a metric to measure the combined "best" score by a logical union of membership functions. This is obtained by finding the de-fuzzified quantity using centroid method. 8



b) Let A be a fuzzy set defined by :

$$\underline{A} = \frac{0{\cdot}5}{x_1} + \frac{0{\cdot}4}{x_2} + \frac{0{\cdot}7}{x_3} + \frac{0{\cdot}8}{x_4} + \frac{1}{x_5} .$$

Find all $\propto$ and string $\propto$ cut sets of $\underline{A}$. 7

5. a) In structural dynamics a particular structure that has been subjected to a shock environment may be in either of the fuzzy sets "damaged" or "undamaged", with a certain degree of membership over the magnitude of the shock input. If there are two crisp sets, functional (F) and non-functional (NF), then a monotone measure would be the evidence that a particular system that has been subjected to shock loading is a member of functional systems or non-functional systems. Given the evidence from two experts shown here for a particular structure, find the beliefs and plausibility for the focal elements. 8

Focal Elements	m_1	m_2	bel_1	bel_2	pl_1	pl_2
F	0·3	0·2				
NF	0·6	0·6				
$F \cup NF$	0·1	0·2				

b) Compare Possibility and Probability Theories ? 7

6. a) Develop a reasonable membership function for a square, based on the geometric properties of a rectangle using the Inference method. 8

b) For controlling speed at a newly constructed bridge a fuzzy system is being developed. A fuzzy $\underline{A}$ on a universe of target speeds is as given below :

A = "speed way over target" = $\left\{\frac{0}{T_0}+\frac{0\cdot 8}{T_{0+5}}+\frac{1}{T_{0+10}}+\frac{0\cdot 8}{T_{0+15}}\right\}$

Another fuzzy set, on a universe of braking pressures, given by :

A = "apply brakes with high force" = $\left\{\frac{0\cdot 3}{10}+\frac{0\cdot 8}{20}+\frac{0\cdot 9}{30}+\frac{1}{40}\right\}$

For the compound proposition, IF speed is "way over target", THEN "apply brakes with high force," find a fuzzy relation using classical implication. 7

7. a) Let A, B be two fuzzy sets defined on universe X, prove that

$|\underline{A}|+|\underline{B}|=|\underline{A}\cup\underline{B}|+|\underline{A}\cap\underline{B}|$. 7

b) In designing a fuzzy washing machine, we define a universe of parameters for defining fabric $X = \{x_1, x_2, x_3\}$ and detergents as $Y = \{y_1, y_2, y_3\}$. Let $\underline{A}$ be fuzzy set defining a fabric given by :

$$\underline{A} = \left\{ \frac{0 \cdot 1}{x_1} + \frac{0 \cdot 9}{x_2} + \frac{0 \cdot 0}{x_3} \right\}$$

And $\underline{B}$ is a fuzzy set defining a detergent given by :

$$\underline{B} = \left\{ \frac{0 \cdot 2}{y_1} + \frac{1}{y_2} + \frac{0 \cdot 1}{y_3} \right\}$$

Find the relation $\underline{R} = \underline{A} \times \underline{B}$. If $\underline{C}$ is another fuzzy set defining another detergent given by

$$\underline{C} = \left\{ \frac{0 \cdot 3}{x_1} + \frac{1 \cdot 0}{x_2} + \frac{0 \cdot 2}{x_3} \right\}$$

Find the control $\underline{S}$ of the washing machine by max-min composition. 8