Name : ....
Roll No. : ....
Invigilator's Signature : ....

CS/M.TECH (CSE)/SEM-1/CSEM-101/2011-12

2011
DISCRETE STRUCTURES

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

Answer any *five* questions.

1. Let $f : R \rightarrow R$ and $g : R$ and $g : R \rightarrow R$ be defined by $f(x) = x^2 + 3$, $g(x) = x + 6$, Then find the *fog* and *gof*. 2

2. Prove that in a group (G, O), $(a^{-1})^{-1} = a \ \forall \ a \in G$. 2

3. Examine whether the following permutation is even or odd. 2

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$$

4. Fine the number of pendant and internal vertices in a binary tree with 7 vertices. 2

5. Prove that there exists no graph with four edges having vertices of degree 4, 3, 2, 1. 2

6. Prove that in a ring $(R, +, .)$ $a.0 = 0.a\ 0$ for all a in R 2

GROUP – B

Answer any *three* questions.

7. Let R be a relation defined on Z × Z by "(a,b) R (c,d)" if and only if $a + d = b + c$ for (a,b), $(c,d) \in Z \times Z$. Show that R is an equivalence relation. 5

8. Let S be set of all real matrices $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a^2 + b^2 = 1 \right\}$.

   Show that S forms a commutative group under matrix multiplication. 5

9. If H and K are two subgroups of the group (G, o) then

   HK = $\{ x \in G : x = hok, h \in H, k \in K \}$

   Then prove that HK is a subgroups of G if and only if HK = KH. 5

10. Prove that the number of odd degree vertices in a graph is always even. 5

11. Find the number of edges in a complete graph with n vertices. 5

GROUP – C

Answer any *three* questions.

12. a) Prove that the order of each subgroup of a finite group is divisor of the order of the group. 5

b) Prove that every cyclic group is an Abelian group. 5

c) Prove that intersection of any two subgroups of a group (G, o) is a subgroup of G. 5

13. a) If A, B, C are any three subsets of a set, then prove that

$A \times (B \cup C) = (A \times B) \cup (A \times C)$ 5

b) Let H be a subgroup of a group G. Then H is normal in G if and only if

$x h x^{-1} \in H \; \forall \; h \in H$ and $\forall \; x \in G$. 5

c) Discuss the nature of the following mapping : $2 \times 2\frac{1}{2} = 5$

i) $f : R \rightarrow R$ defined by $f(x) = x^3 - x$, $x \in R$

ii) $f : R \rightarrow R$ defined by $f(x) = |x|$, $x \in R$

14. a) Assuming that the set E of all real numbers of the form $a + b\sqrt{2}$ with a, b are integers from a ring w.r.t. the ordinary addition and multiplication, show that E is an integral domain. Is it a field ? 5

b) Using generating function solve the recurrence relation

$a_n - 7\,a_{n-1} + 10\,a_{n-2} = 2 \quad \forall\; n > 1$ and $a_0 = 3$, $a_1 = 3$

5

c) Show that $A \times (B - C) = (A \times B) - (A \times C')$ where C' is the complement of C in U where U is a universal set. 5

15. a) Define tree. Prove that a tree with n vertices has $(n - 1)$ edges. 7

b) Using Kruskal's algorithm find the minimal spanning tree of the following graph. 8