



Name : .....  
Roll No. : .....  
Invigilator's Signature : .....

**CS/M.TECH (CSE)/SEM-1/CSEM-101/2011-12**

**2011**

**DISCRETE STRUCTURES**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP – A**

Answer any *five* questions.

1. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2 + 3$ ,  $g(x) = x + 6$ , Then find the  $f \circ g$  and  $g \circ f$ . 2
2. Prove that in a group  $(G, 0)$ ,  $(a^{-1})^{-1} = a \forall a \in G$ . 2
3. Examine whether the following permutation is even or odd. 2  
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$$
4. Find the number of pendant and internal vertices in a binary tree with 7 vertices. 2
5. Prove that there exists no graph with four edges having vertices of degree 4, 3, 2, 1. 2
6. Prove that in a ring  $(R, +, \cdot)$   $a \cdot 0 = 0 \cdot a = 0$  for all  $a$  in  $R$  2



**GROUP – B**

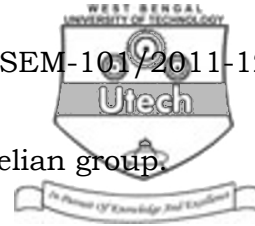
Answer any *three* questions.

7. Let  $R$  be a relation defined on  $Z \times Z$  by “ $(a, b) R (c, d)$ ” if and only if  $a + d = b + c$  for  $(a, b), (c, d) \in Z \times Z$ . Show that  $R$  is an equivalence relation. 5
8. Let  $S$  be set of all real matrices  $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a^2 + b^2 = 1 \right\}$ .  
Show that  $S$  forms a commutative group under matrix multiplication. 5
9. If  $H$  and  $K$  are two subgroups of the group  $(G, o)$  then  
 $HK = \{ x \in G : x = hok, h \in H, k \in K \}$   
Then prove that  $HK$  is a subgroups of  $G$  if and only if  $HK = KH$ . 5
10. Prove that the number of odd degree vertices in a graph is always even. 5
11. Find the number of edges in a complete graph with  $n$  vertices. 5

**GROUP – C**

Answer any *three* questions.

12. a) Prove that the order of each subgroup of a finite group is divisor of the order of the group. 5

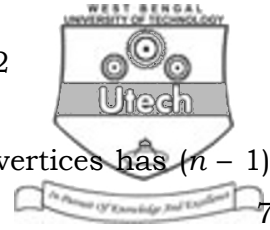


- b) Prove that every cyclic group is an Abelian group. 5
- c) Prove that intersection of any two subgroups of a group  $(G, o)$  is a subgroup of  $G$ . 5
13. a) If  $A, B, C$  are any three subsets of a set, then prove that  

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$
 5
- b) Let  $H$  be a subgroup of a group  $G$ . Then  $H$  is normal in  $G$  if and only if  

$$x h x^{-1} \in H \quad \forall h \in H \text{ and } \forall x \in G.$$
 5
- c) Discuss the nature of the following mapping :  $2 \times 2 \frac{1}{2} = 5$
- i)  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^3 - x, x \in \mathbb{R}$
- ii)  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = |x|, x \in \mathbb{R}$
14. a) Assuming that the set  $E$  of all real numbers of the form  $a + b\sqrt{2}$  with  $a, b$  are integers from a ring w.r.t. the ordinary addition and multiplication, show that  $E$  is an integral domain. Is it a field ? 5
- b) Using generating function solve the recurrence relation  

$$a_n - 7a_{n-1} + 10a_{n-2} = 2 \quad \forall n > 1 \text{ and } a_0 = 3, a_1 = 3$$
 5
- c) Show that  $A \times (B - C) = (A \times B) - (A \times C')$  where  $C'$  is the complement of  $C$  in  $U$  where  $U$  is a universal set. 5



15. a) Define tree. Prove that a tree with  $n$  vertices has  $(n - 1)$  edges. 7
- b) Using Kruskal's algorithm find the minimal spanning tree of the following graph. 8

