

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS / M.TECH (CSE) SEM-1 / CSEM-101 / 2010-11**

**2010-11**

**DISCRETE STRUCTURE**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words  
as far as practicable.*

**GROUP – A**

**( Very Short Answer Type Questions )**

Attempt any *five* of the following questions :  $5 \times 2 = 10$

1.
  - i) Find the number of vertices in a graph with 15 edges if each vertex has degree 2.
  - ii) Find the number of pendant and internal vertices in a binary tree having 7 vertices.
  - iii) Show that the number of edges in a complete graph with  $n$  vertices is  $\frac{n(n-1)}{2}$ .
  - iv) If  $u + 3x = 5$ ,  $2y - v = 7$  and correlation coefficient of  $x$  and  $y$  is 0.12, find the correlation coefficient of  $u$  and  $v$ .
  - v) Examine whether the following permutation is even or odd :  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$ .



- vi) Prove that in a Ring  $(R, +, \cdot)$   
 $a \cdot 0 = 0 \cdot a = 0$  for all  $a$  in  $R$ .

**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* of the following.  $3 \times 5 = 15$

2. Prove that a tree with  $n$  vertices has  $(n - 1)$  edges.
3. Draw the graph / digraph whose adjacency matrix is

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

4. Show that the number of odd degree vertices in a graph is always even.
5. Show that  $A \times (B - C) = (A \times B) - (A \times C')$  where  $C'$  is the complement of  $C$  in  $U$  where  $U$  is a universal set.
6. Using generating function solve the recurrence relation

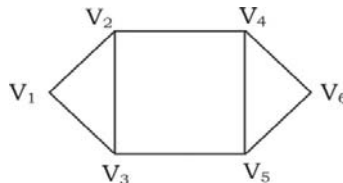
$$a_n - 7a_{n-1} + 10a_{n-2} = 2 \quad \forall n > 1 \text{ and } a_0 = 3, a_1 = 3.$$

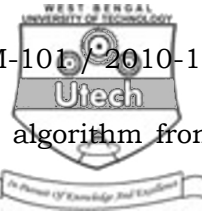
**GROUP – C**

**( Long Answer Type Questions )**

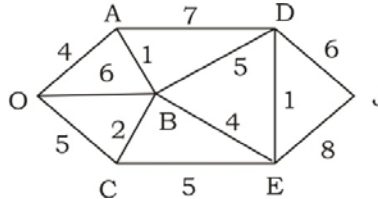
Answer any *three* of the following.  $3 \times 15 = 45$

7. a) Define complement of a graph. Draw the complement of the following graph.



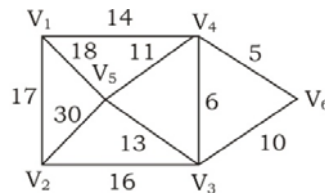


- b) Find the shortest path using Dijkstra's algorithm from the vertex  $O$  to  $J$  in the following graph.

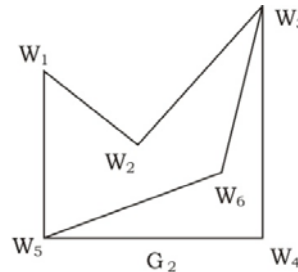
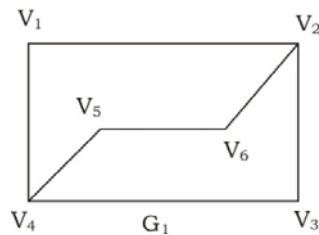


7 + 8

8. a) Find the minimal spanning tree of the following graph.

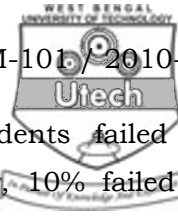


- b) Show that the graphs  $G_1$  and  $G_2$  are isomorphic.



8 + 7

9. a) For two variables  $x$  and  $y$  the equations of two regression lines are  $x + 4y + 3 = 0$  and  $4x + 9y + 5 = 0$ . Identify which one is 'of  $y$  on  $x$ '. Find the means of  $x$  and  $y$ . Find the correlation coefficient between  $x$  and  $y$ . Estimate the value of  $x$  when  $y = 1.5$ .
- b) If  $X$  is a Binomial variate with parameters ' $n$ ' and ' $p$ ' then prove that mean is greater than variance. 8 + 7



10. a) In a certain class 25% of the students failed in Mathematics, 15% failed in Chemistry, 10% failed in both Mathematics and Chemistry. A student is selected at random.

- i) If he failed in Mathematics, what is the probability that he failed in Chemistry ?
- ii) If he failed in Chemistry, what is the probability that he failed in Mathematics?

- b) The distribution function  $F(x)$  of a variate  $X$  is defined as follows

$$\begin{aligned} F(x) &= A, & \infty < x < -1 \\ &= B, & -1 \leq x < 0 \\ &= C, & 0 \leq x < 2 \\ &= D, & 2 \leq x < \infty \end{aligned}$$

where  $A, B, C, D$  are constants. Determine the values of  $A, B, C, D$ , given that  $p(X=0)=1/6$  and  $p(X>1)=2/3$ .

11. a) Let  $S$  be the set of all real matrices

$$\left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a^2 + b^2 = 1 \right\}. \text{ Show that } S \text{ forms a}$$

commutative group under matrix multiplication.

- b) Assuming that the set  $E$  of all real numbers of the form  $a + b\sqrt{2}$  with  $a, b$  are integers form a ring w.r.t. the ordinary addition and multiplication. Show that  $E$  is an integral domain. Is it a field ?

- c) Prove that the order of each subgroup of a finite group is a divisor of the order of the group.

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