*Name :* .....

*Roll No. :* .....

*Invigilator's Signature :* .....

**CS/M.Tech(CST)/SEM-1/CST-1103A7/2012-13**

# 2012
# ALGORITHM AND COMPLEXITY

*Time Allotted :* 3 Hours *Full Marks :* 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A

## ( Multiple Choice Type Questions )

1. Choose the correct alternatives of the following : $10 \times 1 = 10$

   i) A machine needs a minimum of 100s to sort 1000 names by quick sort. The minimum time needed to sort 100 names will be approximately

   a) 50·2s b) 6·7s

   c) 72·7s d) 11·2s.

   ii) Given 2 sorted lists of size $m$ and $n$ respectively. Number of comparisons needed in the worst case by the merge sort algorithm will be

   a) $mn$ b) max $(m, n)$

   c) min $(m, n)$ d) $m + n - 1$.

iii) The maximum number of comparisons needed to sort 7 items using radix sort is (assume each item is a 4 digit decimal number)

a) 280 b) 40

c) 47 d) 38.

iv) The running time of Kruskal's algorithm for MST is

a) $O(E)$ b) $O(V)$

c) $O(E \log V)$ d) none of these.

v) The running time of Floyd-Warshall algorithm is

a) $\theta(n)$ b) $\theta(n * n)$

c) $\theta(n^3)$ d) $\theta(n \log n)$.

vi) Identify the correct statements about DFS traversal.

(I) It can be used to determine whether a graph is a cyclic or not.

(II) It identifies the connected component of an undirected graph.

(III) Traverses a single path of the graph until it visits a node with no successor.

a) Both (I) and (III) are correct

b) Both (II) and (III) are correct

c) Both (I) and (II) are correct

d) All are correct.

vii) Consider an undirected graph $G$ with $n$ vertices and $e$ edges. What is the time taken by DFS if the graph is represented by (i) Adjacency matrix and (ii) Adjacency list ?

a) $O(n * n)$, $O(n)$

b) $O(n * n)$, $O(e)$

c) $O(e)$, $O(n * n)$

d) $O(e + n)$, $O(e)$.

viii) The recurrence relation, $T(n) = m\,T(n/2) + an^2$ is satisfied by

a) $T(n) = O(n^m)$

b) $T(n) = O(n \log m)$

c) $T(n) = O(n \log n)$

d) $T(n) = O(m \log n)$.

ix) Let $T(n)$ be the number of different binary search trees on $n$ distinct elements then

$T(n) = \sum T(k-1)T(x)$, where $x$ is

a) $n - k + 1$ b) $n - k$

c) $n - k - 1$ d) $n - k - 2$.

x) Consider the following three claims :

(I) $(n + k)^m = (n^m)$, where $k$ and $m$ are constants

(II) $2^{n+1} = O(2^n)$

(III) $2^{2n+1} = O(2^n)$

Which of the following claims are correct ?

a) I and II b) I and III

c) II and III d) I, II and III.

## GROUP – B

### ( Short Answer Type Questions )

Answer any *three* of the following. $3 \times 5 = 15$

2. Write down the algorithm for matrix chain multiplication using dynamic programming. Compute the time complexity of your algorithm.

3. Prove that, if any *NP*-complete problem belongs to class *P*, then *P* = *NP*.

4. Show that in fractional Knapsack, if objects are selected in order of decreasing $\frac{V_i}{W_i}$, the Greedy method finds an optimal solution.

5. For any two functions $f(n)$ and $g(n)$, prove that,

   $\max(f(n), g(n)) = \theta(f(n) + g(n))$

6. Design an algorithm for the $n$-colouring problem, using back tracking technique.

## GROUP – C

### ( Long Answer Type Questions )

Answer any *three* of the following. $3 \times 15 = 45$

7. You are given a set of $n$ jobs. Associated with each job $j$ is a processing time $t_j$ and a deadline $d_j$ by which it must be completed. A feasible schedule is a permutation of the jobs such that if the jobs are processed in that order, then each job finishes by its deadline. Define a greedy schedule to be one in which the jobs are processed in non-decreasing order of deadlines. Show that if there exists a feasible schedule, then all greedy schedules are feasible. Also show that greedy algorithm for job scheduling with deadline always gives an optimal solution.

   $2 + 5 + 8$

8. Show that the running time of QUICK-SORT is $(n^2)$ when the array contains distinct elements and is sorted in decreasing order. Show that QUICK-SORT's best case running time is $\Omega(n \log n)$. 5 + 10

9. Write down the differences between Dynamic programming and Divide and conquer. What are the characteristics of Dynamic Programming ? The distance matrix between the four houses ( $A$, $B$, $C$ and $D$ ) is given below :

$$\begin{bmatrix} 0 & 2 & 9 & 10 \\ 1 & 0 & 6 & 4 \\ 15 & 7 & 0 & 8 \\ 6 & 3 & 12 & 0 \end{bmatrix}$$

A newspaper agent daily drops the newspaper to the house assigned in such a manner that he has to cover all the houses with minimum travel cost. Compute the path starting from house $A$ using Dynamic Programming. 2 + 3 + 10

10. Is P = NP ? Given a graph *G*, does this graph G contain a Hamiltonian cycle ? That means whether there exists a simple cycle such that each vertex is visited only once other than the starting vertex or not. Prove that Hamiltonian cycle is in NP.

    Given a Boolean circuit *B* as an input with a single node, a Circuit-SAT problem deals whether there is an assignment of values to the circuit's inputs so that its output value is 1. Such value assignments is referred to as satisfying assignment. Prove that Circuit SAT in NP. Also show that Circuit-SAT is NP-Complete. 2 + 5 + 4 + 4

11. Define *N*-Queens problem. Write an algorithm to find all the possible solutions to place the 8 queens for the 8 × 8 chessboard. Draw the state space for 4-queens problem.

    2 + 8 + 5