Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/M.TECH (CSE)/SEM-1/PGCS(MCE)-101/2011-12

2011
ADVANCED ENGINEERING MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words
as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer the following questions : $7 \times 2 = 14$

   a) Evalute $\oint_c \dfrac{dz}{z - \alpha}$, where c denote simple closed

   rectifiable curve & α is an interior point of *C*.

   b) Let $f(z)$ be analytic in a domain *D*, then show that *f* is constant in *D* if $I_m\{f(z)\}$ is constant in *D*.

   c) Show that $\Delta \log f(x) = \log \left\{ 1 + \dfrac{\Delta f(x)}{f(x)} \right\}$

   d) If *A*, *B*, *C* be mutually independent event, then prove that *A* & *B* + *C* are independent.

   e) Define extreme point of a function of two variables.

   f) Show that the probability of occurrence of only one of the event *A* & *B* is $P(A) + P(B) - 2P(AB)$.

   g) Prove that $P(a < \times \le b) = F(b) - F(a)$.

40032   [Turn over

2. a) Construct Lagrange's interpolation polynomial by using the following data :

x : 40 45 50 55

$y = f(x)$: 15·22 13·99 12·62 11·13
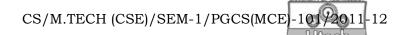
Hence find $f(53)$ 4 + 2

b) Salve any one of the following difference equations: 4 + 4
   i) $u_{x+1} - u_x = x^2 + 3x$
   ii) $u_{x+2} - 11\, u_{x+1} + 30\, u_x = \cos 2\, x$
   iii) $u_{x+2} - 4\, u_x = 9\, x^2$.

3. a) State Cauchy Residue theorem & evaluate

$$\oint_{|z|=2} \frac{z\, d\, z}{z^4 - 1}$$ 2 + 4

b) Show that the transformation $w = \dfrac{2z + 3}{z - 4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the straight line $4\, u + 3 = 0$

     4

c) Find the bilinear transformation that maps the points 0, 1, 2 of the z–plane onto -1, i, 1 of w–plane.   4

4. a) Show that $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{1 - 2a \cos\theta + a^2}\, d\theta = \frac{2\pi a^2}{1 - a^2}, (a^2 < 1)$.   6

b) Use Runge – Kutta method of order two to find $y\,(0·2)$ & $y\,(0·4)$ given that $y\,\dfrac{dy}{dx} = y^2 - x$, $y\,(0) = 2$, by taking $h = 0·2$.   8

5. a) State & prove Euler's theorem (2nd order) for homogeneous function of several variables.   2 + 4

b) If $u = f(\, y - z,\, z - x,\, x - y\,)$, prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$ 4

c) If $u = \cos^{-1} \left\{ \dfrac{x + y}{\sqrt{x} + \sqrt{y}} \right\}$, then prove that

$x\, u_x + y\, u_y + \dfrac{1}{2} \cot u = 0.$ 4

6. a) There are two identical urns containers respectively 4 white, 3 red balls & 3 white, 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probabitity that it is from the first urn ? 7

   b) Find the maxima & minima of the function $x^3 + y^3 - 3\,x - 12\,y + 20$. Find also the saddle points if exists. 7

7. a) Investigate the continuity of the following functions at the given points :

   i) $f(x, y) = (x^2 + y^2) \sin \dfrac{1}{x^2 + y^2}$, $(x, y) \neq (0, 0)$

       $= 0$, $(x, y) = (0, 0)$

       at $(0, 0)$. 5

   ii) $f(x, y) = \dfrac{3\,x\,y}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$

       $= 0$, $(x, y) = (0, 0)$

       at $(0, 0)$. 5

   b) State the geometrical interpretation of partial derivative of a function of two independent variables. 4