	<u>Uteah</u>
Name:	
Roll No.:	" Dearly Complete 2nd Explana
Invigilator's Signature :	

CS/M.Tech(CSE)/SEM-1/CST-611/2012-13 2012

ADVANCED ENGINEERING & MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer question no. 1 and any five from the rest taking at least two from Group - B and Group - C.

GROUP - A

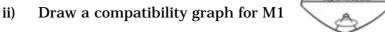
- 1. a) Define cyclic group. Prove that "Every cyclic group is abelian but reverse is not true". 5
 - b) A function $f:(R,.) \rightarrow (R,.)$ and define that $f(x) = x^2$, Test whether f is Isomorphism or not. 5
 - c) Consider the following machine M_1 :

PS	NS, Z			
	I_1	I_2	I_3	I_4
A	_	C, 1	E, 1	B, 1
В	<i>E</i> , 0	F, 1		_
C	F, 0	F, 1		_
D	_	1	<i>B</i> , 1	_
E	_	F, 0	A, 0	D, 1
F	<i>C</i> , 0	_	В, 0	<i>C</i> , 1

i) Construct a Merger table for M1

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- iii) Obtain a closed covering of M1
- iv) Construct a minimized machine M1* of M1.



GROUP - B

- 2. Construct a Mealy machine to detect the squence 1010 from a long random sequence of 0 and 1 where overlapping sequences are accepted. Convert it to Moore machine. 6+4
- 3. Show that the language $\binom{n}{n}$ [set of balanced parenthesis] is not regular. From the identities of regular expression prove that the following three are equivalent:
 - a) (011((11) * + (01) *) *) *011
 - b) 011(((1+0)1)*011)*
 - c) 011(((11) *(01) *) *011) * 5 + 5
- 4. Prove that Context Free Languages are closed under concatenation. Construct an equivalent PDA for the following Context Free Grammar:

$$S \to aA$$

$$A \to aABC/bB/a$$

$$C \to c$$

Convert the following grammar into GNF:

$$S \rightarrow AA/a$$

 $A \rightarrow SS/b$. $2 + 3 + 5$

5. Design a turing machine to perform 2's complement operation on binary string. Show IDs for the string 010 and 101. Prove that the problem "Language generated by a turing machine is empty" is undecidable. 5+5

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GROUP - C

- 6. a) Prove that "Every group of prime order is cyclic". If the order of the groups are 15, 21, 35, then identify the cyclic group.
 - b) Define integral domain. Prove that "A field is an integral domain". 5+5
- 7. a) Solve the recurrence relation $a_n 2a_{n-1} = 3^n$, $a_1 = 5$.
 - b) Find a formula for the general term F_n of the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, ... 3+7
- 8. a) Find the disjunctive normal forms of the following Boolean expression by (i) truth table method and (ii) algebraic method:

$$f(x, y, z) = xy + yz^{\prime}$$

b) Use the method of generating function to solve the recurrence relation :

$$a_n = 4a_{n-1} - 4a_{n-2} + 4^n$$
, $n \ge 2$, $a_0 = 2$, $a_1 = 8$.

5 + 5

- 9. a) Define Dihedral (D_4) group and permutation group. Why D_4 is not cyclic group ? If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ & & & & & \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$, then what is the value of σ^{100} ?
 - b) The set { 1, 2, 4, 7, 8, 11, 13, 14 } is a group under modulo 15. Then find the inverse of 4 & 7.

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