Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/M.Tech(CSE)/SEM-1/CST-611/2012-13

2012

ADVANCED ENGINEERING & MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer question no. 1 and any *five* from the rest taking at least *two* from ***Group – B*** and ***Group – C***.

GROUP – A

1. a) Define cyclic group. Prove that "Every cyclic group is abelian but reverse is not true". 5

   b) A function $f : (R, .) \rightarrow (R, .)$ and define that $f(x) = x^2$, Test whether f is Isomorphism or not. 5

   c) Consider the following machine M_1 :

PS	NS, Z			
	I_1	I_2	I_3	I_4
A	—	C, 1	E, 1	B, 1
B	E, 0	F, 1	—	—
C	F, 0	F, 1	—	—
D	—	—	B, 1	—
E	—	F, 0	A, 0	D, 1
F	C, 0	—	B, 0	C, 1

   i) Construct a Merger table for M1

ii) Draw a compatibility graph for M1

iii) Obtain a closed covering of M1

iv) Construct a minimized machine M1* of M1.

3 + 2 + 2 + 3

GROUP – B

2. Construct a Mealy machine to detect the squence 1010 from a long random sequence of 0 and 1 where overlapping sequences are accepted. Convert it to Moore machine. 6 + 4

3. Show that the language $(\ )^n$ [set of balanced parenthesis] is not regular. From the identities of regular expression prove that the following three are equivalent :

   a) (011((11) * + (01) *) *) *011

   b) 011(((1 + 0)1) *011) *

   c) 011(((11) *(01) *) *011) * 5 + 5

4. Prove that Context Free Languages are closed under concatenation. Construct an equivalent PDA for the following Context Free Grammar :

   $S \rightarrow aA$

   $A \rightarrow aABC/bB/a$

   $C \rightarrow c$

   Convert the following grammar into GNF :

   $S \rightarrow AA/a$

   $A \rightarrow SS/b.$ 2 + 3 + 5

5. Design a turing machine to perform 2's complement operation on binary string. Show IDs for the string 010 and 101. Prove that the problem "Language generated by a turing machine is empty" is undecidable. 5 + 5

GROUP – C

6. a) Prove that "Every group of prime order is cyclic". If the order of the groups are 15, 21, 35, then identify the cyclic group.

   b) Define integral domain. Prove that "A field is an integral domain". 5 + 5

7. a) Solve the recurrence relation

   $a_n - 2a_{n-1} = 3^n,\ a_1 = 5.$

   b) Find a formula for the general term F_n of the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, ... 3 + 7

8. a) Find the disjunctive normal forms of the following Boolean expression by (i) truth table method and (ii) algebraic method :

   $f(x, y, z) = xy + yz'$

   b) Use the method of generating function to solve the recurrence relation :

   $a_n = 4a_{n-1} - 4a_{n-2} + 4^n,\quad n \geq 2,\ a_0 = 2,\ a_1 = 8.$

   5 + 5

9. a) Define Dihedral (D_4) group and permutation group. Why D_4 is not cyclic group ? If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$, then what is the value of σ^{100} ?

   b) The set { 1, 2, 4, 7, 8, 11, 13, 14 } is a group under modulo 15. Then find the inverse of 4 & 7. 7 + 3