



Name :

Roll No. :

Invigilator's Signature :

CS/ME(CSE)/M.Tech(SE)/SEM-1/PGCSE-104-A/PGSE-104/2009-10

2010

ADVANCED ALGORITHMS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

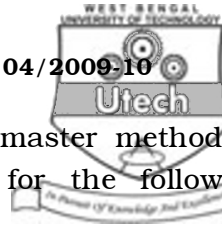
Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be provided by the Institution.

Answer Question No. 1 and any four from the rest.

1. Comment critically for the following statements and justify your opinion any four. $4 \times 3\frac{1}{2} = 14$
 - a) All NP complete problems are NP-Hard as well, but the reverse is not true.
 - b) Exhaustively checking all parenthesizations for the matrix-chain multiplication problem does not lead to an efficient algorithm.
 - c) $O(n)$ is always better than $O(n)^2$.
 - d) The dfs of an undirected graph always gives only two types of edges.
 - e) It is impossible to place 3-queens in a 3×3 chess-board such that no two-queens attack each other.
2.
 - a) Find a recurrence for the running time of the following recursive insertion sort : In order to sort $A[1..n]$, we recursively sort $A[1..n-1]$ and then insert $A[n]$ into the sorted array $A[1..n-1]$.
 - b) Prove that $n! = \omega(2^n)$ and $n! = o(n^n)$.
 - c) Find the solution to $T(n) = 2T(\lfloor n/2 \rfloor) + 17$.

4 + 6 + 4

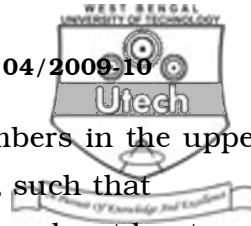


3. a) State the master theorem. Use the master method to find the tight asymptotic bounds for the following recurrences :
- i) $T(n) = 3T(n/2) + n \lg n$
- ii) $T(n) = 9T(n/3) + n.$
- b) Write a pseudocode for linear search with the following conditions :
- Input : A sequence of n numbers $A = \{ a_1, a_2, \dots, a_n \}$ and value v .
- Output : An index i such that $v = A[i]$ or NIL if v is not in A . 4 + 2 + 2 + 6
4. a) Prove that in a depth-first search of an undirected graph G , every edge of G is either a tree edge or a back edge.
- b) What is the shortest path from the root to a leaf node in a decision tree describing any algorithm that correctly sorts an arbitrary list on n elements by comparison of keys ? Explain your answer. 6 + 8
5. a) Classify different types of edges in the depth-first forest.
- b) You are given a set of boxes to be packed into bins. All the boxes have the same width and depth (the same as the width and depth of the bins), but they have different heights. The heights are given in a list $H = (h_1 \dots, h_n)$. The goal is to pack the boxes in bins using as few bins as possible. Suggest a greedy algorithm for this problem.
- c) Define topological sort on a digraph. Show the ordering of vertices produced by topological sort when it is run on the following dag. 4 + 6 + 4



6. a) Give a simple example of a directed graph with negative weight edge for which Dijkstra's algorithm produce incorrect result.
- b) Give an algorithm for listing the vertices of one of the cycles of a graph with negative weight cycles. 4 + 6
7. a) Perform a breath first search on the following graph and write the labels in the order you visit them. What is the maximum size of the queue during this BFS run ?

- b) Let S be a sequence $\langle x_1, x_2, \dots, x_n \rangle$ of numbers in any order. Prove constructively that any algorithm that



selects a number from the set of numbers in the upper half of S (i.e. any number x_i , $1 \leq i \leq n$, such that $x_i \geq$ the upper median m of S) must make at least $\underline{\Omega} n/2 \rightarrow$ comparisons. 7 + 7

8. a) What is the single source shortest path problem ? Explain Bellman-Ford algorithm using 1 as source vertex in the following graph.

- b) Give an algorithm for matrix chain multiplication problem using dynamic approach. 2 + 6 + 6

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