Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/ME(CSE)/M.Tech(SE)/SEM-1/PGCSE-104-A/PGSE-104/2009-10

# 2010

# ADVANCED ALGORITHMS

*Time Allotted* : 3 Hours  *Full Marks* : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Graph sheet(s) will be provided by the Institution.*

Answer Question No. 1 and any *four* from the rest.

1. Comment critically for the following statements and justify your opinion any *four*. $4 \times 3\frac{1}{2} = 14$

   a) All NP complete problems are NP-Hard as well, but the reverse is not true.

   b) Exhaustively checking all parenthesizations for the matrix-chain multiplication problem does not lead to an efficient algorithm.

   c) $O(n)$ is always better than $O(n)^2$.

   d) The *dfs* of an undirected graph always gives only two types of edges.

   e) It is impossible to place 3-queens in a $3 \infty 3$ chess-board such that no two-queens attack each other.

2. a) Find a recurrence for the running time of the following recursive insertion sort : In order to sort $A[1..n]$, we recursively sort $A[1..n-1]$ and then insert $A[n]$ into the sorted array $A[1..n-1]$.

   b) Prove that $n! = \omega(2^n)$ and $n! = o(n^n)$.

   c) Find the solution to $T(n) = 2T([n/2]) + 17$.

   4 + 6 + 4

3. a) State the master theorem. Use the master method to find the tight asymptotic bounds for the following recurrences :

   i) $T(n) = 3T(n/2) + n \lg n$

   ii) $T(n) = 9T(n/3) + n$.

   b) Write a pseudocode for linear search with the following conditions :

   Input : A sequence of $n$ numbers $A = \{ a_1, a_2, \ldots, a_n \}$ and value $v$.

   Output : An index $i$ such that $v = A\{i\}$ or NIL if $v$ is not in $A$. 4 + 2 + 2 + 6

4. a) Prove that in a depth-first search of an undirected graph $G$, every edge of $G$ is either a tree edge or a back edge.

   b) What is the shortest path from the root to a leaf node in a decision tree describing any algorithm that correctly sorts an arbitrary list on $n$ elements by comparison of keys ? Explain your answer. 6 + 8

5. a) Classify different types of edges in the depth-first forest.

   b) You are given a set of boxes to be packed into bins. All the boxes have the same width and depth ( the same as the width and depth of the bins ), but they have different heights. The heights are given in a list $H = (h_1 \ldots, h_n)$. The goal is to pack the boxes in bins using as few bins as possible. Suggest a greedy algorithm for this problem.

   c) Define topological sort on a diagraph. Show the ordering of vertices produced by topological sort when it is run on the following dag. 4 + 6 + 4

6. a) Give a simple example of a directed graph with negative weight edge for which Dijkstra's algorithm produce incorrect result.

   b) Give an algorithm for listing the vertices of one of the cycles of a graph with negative weight cycles. 4 + 6

7. a) Perform a breath first search on the following graph and write the labels in the order you visit them. What is the maximum size of the queue during this BFS run ?

   b) Let $S$ be a sequence $< x_1 , x_2 , \ldots , xn >$ of numbers in any order. Prove constructively that any algorithm that

selects a number from the set of numbers in the upper half of $S$ ( i.e. any number $xi$, $1 \leq i \leq n$, such that $xi$ = > the upper median $m$ of $S$ ) must make at least $\underline{\Omega}\, n/2 \rightarrow$ comparisons. 7 + 7

8. a) What is the single source shortest path problem ? Explain Bellman-Ford algorithm using 1 as source vertex in the following graph.

b) Give an algorithm for matrix chain multiplication problem using dynamic approach. 2 + 6 + 6